Exercise AP-331

Evaluating a road project accounting for DWL in raising tax revenue

The Economic Skills Project

1 Problem

Problem

A government would like to reduce traffic congestion by building a new highway. Construction would cost $30 million per year for 5 years (years 1-5). The money to pay for it would be raised via a tax with a deadweight loss of $0.20 per dollar of revenue. Beginning in year 6, the road would be free and would provide $17 million in benefits to drivers every year for 20 years (years 6-25). What is the NPV of the project at an interest rate of 5%? Round your answer to the nearest million and account for the cost of raising the funds.

2 Answer

Answer

Here's the solution:

- $10 million

3 Method

Solution method

Here's one approach:

1. Calculate the cost of construction accounting for the DWL.
2. Draw the cash flow diagram.
3. Split the cash flows into two groups: costs and benefits.
4. Use the finite stream PV formula for the costs.
5. Use the finite stream formula in year 5 to find a lump sum benefit.
6. Take the present value of the year-5 lump sum.
7. Take the difference to find the NPV.

4 Solution

4.1 Step 1

Calculate the cost of construction

Although the government is paying the builder \$30 million, the true cost to its citizens is larger due to the DWL arising from the tax. Since that's \$0.20 per dollar of revenue, on \$30 million it would be:

$$\text{DWL} = \$30\text{M} \cdot 0.20 = \$6\text{M}$$

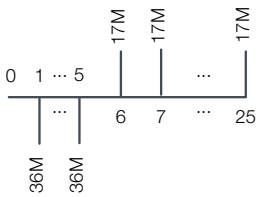
The true cost per year is thus:

$$\$30\text{M} + \$6\text{M} = \$36\text{M}$$

4.2 Step 2

Draw the cash flow diagram

Here's how it looks:

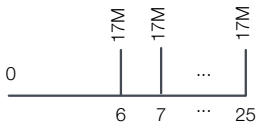


4.3 Step 2

Split the flows into costs and benefits

Costs:

Benefits:



4.4 Step 4

Use the finite stream PV formula for the costs

The present value of an finite stream of identical payments F starting at time 1 and ending at T when the interest rate is r is given by:

$$PV = \frac{F}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

Filling in the other numbers and calculating the PV of the costs, PVC, gives:

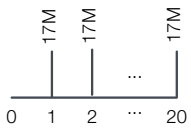
$$PVC = \frac{\$36M}{0.05}\left(1 - \frac{1}{1.05^5}\right)$$

$$PVC = \$156M$$

4.5 Step 5

Use the finite stream formula in year 5

From year 5's perspective the benefit stream looks like $17 million a year starting in 1 year and running for 20 years:



Use the finite stream formula in year 5, continued

The present value of a finite stream of identical payments F starting at time $T+1$ when the interest rate is r is given by:

$$PV = \frac{F}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

Filling in the other numbers and calculating the year-5 lump sum equivalent of the benefits, PVB_5, gives:

$$PVB_5 = \frac{\$17M}{0.05}\left(1 - \frac{1}{1.05^{20}}\right)$$

$$PVB_5 = \$212M$$

4.6 Step 6

Take the PV of the year-5 lump sum

The PV of the benefits in year 0 is just the PV of the year-5 lump sum:

$$PVB = \frac{PVB_5}{1.05^5} = \$166M$$

4.7 Step 7

Take the difference to find the NPV

Armed with the previous results, the NPV is straightforward:

$$\text{NPV} = \text{PVB} - \text{PVC}$$

$$\text{NPV} = \$166M - \$156M$$

$$\text{NPV} = \$10M$$

Done!