Exercise CD-101

Deriving demand curves for perfect complements preferences

The Economic Skills Project

1 Problem

Problem

An individual regards goods X and Y as perfect complements and likes to have 5 units of Y for each unit of X. What are the individual's demand curves for the two goods?

2 Answer

Answer

Here are the demand curves:

$$X = \frac{M}{P_X + 5P_Y}$$
$$Y = \frac{5M}{P_X + 5P_Y}$$

3 Method

Solution method

Here's one approach:

1. Solve for an equation linking the individual's preferred X and Y.
2. Use it to solve the budget constraint for the demand for X.
3. Use that and the equation from 1 to find the demand for Y.
4. Summarize the results.

4 Solution

4.1 Step 1

Solve for an equation linking X and Y

Since the individual wants 5 units of Y for each unit of X their preferred ratio of the two goods is:

$$\frac{Y}{X} = \frac{5}{1}$$

Multiplying both sides by X:

$$Y = 5X$$

4.2 Step 2

Solve the budget constraint for the demand for X

The budget constraint is:

$$P_X X + P_Y Y = M$$

Using the equation from step 1 to eliminate Y:

$$P_X X + P_Y (5X) = M$$

Collecting terms in X:

$$(P_X + 5P_Y) X = M$$

Solving for X gives the demand equation:

$$X = \frac{M}{P_X + 5P_Y}$$

4.3 Step 3

Find the demand for Y

From step 1 the amount of Y given X will be:

$$Y = 5X$$

Substituting in the demand for X gives the demand for Y:

$$Y = 5\left(\frac{M}{P_X + 5P_Y}\right)$$

Rewriting it slightly:

$$Y = \frac{5M}{P_X + 5P_Y}$$

4.4 Step 4

Summary

Collecting the two equations gives the finished set of demands:

$$X = \frac{M}{P_X + 5P_Y}$$

$$Y = \frac{5M}{P_X + 5P_Y}$$

Done!