Exercise CW-251

Income and substitution effects for Cobb-Douglas preferences

The Economic Skills Project

1 Problem

Problem

A household has the Cobb-Douglas utility function, demand curves, and expenditure function shown below. Initially, $P_X = \$10$, $P_Y = \$20$, and $M = \$2,000$. The government is considering a policy that would raise the price P_X to \$20. There would be no change in M or P_Y. What are the income and substitution effects of the policy?

$$U = X^{0.25} \cdot Y^{0.75}, \ \ M = U\left(\frac{P_X}{0.25}\right)^{0.25}\left(\frac{P_Y}{0.75}\right)^{0.75}$$

$$X = \frac{0.25M}{P_X}, \quad Y = \frac{0.75M}{P_Y}$$

2 Answer

Answer

Here's the solution:

- Substitution effect: -20.3 units of X
- Income effect: -4.7 units of X

3 Method

Solution method

Here's one approach:

1. Use the demand equations to compute X_1 and Y_1.
2. Use the utility function to compute U_1.
3. Use the demand equation to compute X_2.
4. Use the expenditure function to compute M_3.
5. Use M_3 to compute X_3.
6. Compare X_3 to X_1 and X_2 to compute the effects.
7. Check the result.

4 Solution

4.1 Step 1

Use the demand equations to compute X_1 and Y_1

Inserting the initial values of M_1, P_{X1}, and P_{Y1} into the demands gives:

$$X_1 = \frac{0.25 \cdot \$2,000}{\$10} = 50$$

$$Y_1 = \frac{0.75 \cdot \$2,000}{\$20} = 75$$

4.2 Step 2

Use the utility function to compute U_1

Using X_1 and Y_1 to compute U_1:

$$U_1 = 50^{0.25} \cdot 75^{0.75} = 67.77$$

4.3 Step 3

Use the demand equation to compute X_2

Inserting the new values of M_2 (unchanged), P_{X2}, and P_{Y2} (unchanged) into the demands gives:

$$X_1 = \frac{0.25 \cdot \$2,000}{\$20} = 25$$

4.4 Step 4

Use the expenditure function to compute M_3

Inserting U_1 and P_{X2} and P_{Y2} into the expenditure function gives M_3, the expenditure needed to get the original utility at the new prices:

$$M_3 = U_1 \left(\frac{P_{X2}}{0.25}\right)^{0.25} \left(\frac{P_{Y2}}{0.75}\right)^{0.75}$$

$$M_3 = 67.77 \left(\frac{\$20}{0.25}\right)^{0.25} \left(\frac{\$20}{0.75}\right)^{0.75} = \$2,378$$

4.5 Step 5

Use M_3 **to compute** X_3

Inserting M_3 and P_{X2} into the demand equation gives X_3, the amount the household would consume if it had been compensated and remained on the original indifference curve:

$$X_3 = \frac{0.25 \cdot \$2,378}{\$20} = 29.7$$

4.6 Step 6

Compare X_3 **to** X_1 **and** X_2 **to compute the effects**

The substitution and income effects, ΔX_S and ΔX_I, are defined as follows:

$$\Delta X_S = X_3 - X_1$$

$$\Delta X_I = X_2 - X_3$$

Inserting the values of X_1, X_2, and X_3:

$$\Delta X_S = 29.7 - 50 = -20.3$$

$$\Delta X_I = 25 - 29.7 = -4.7$$

4.7 Step 7

Check the result

The sum of the income and substitution effects, $\Delta X_S + \Delta X_I$, should be equal to the actual change in X. The actual change is:

$$X_2 - X_1 = 25 - 50 = -25$$

The sum of the income and substitution effects is:

$$\Delta X_S + \Delta X_I = -20.3 - 4.7 = -25$$

Everything checks - done!

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