Department of Economics
The University of Texas at Austin

Comprehensive Exam
Computational Economics
January 2003

This exam is divided into three parts. Part A is required for everyone. Parts B and C are alternatives: Part B is required if you took Prof. Wilcoxen's course on CGE models and Part C is required if you took the Stochastic Control for Economic Models reading course from Prof. Kendrick as a substitute for Prof. Wilcoxen's course.

Part A

Everyone should complete this section.

Question 1. Please answer *either* part (a) or part (b).

(a) Consider the Chakravarty single sector growth model (or its more recent form as the Taylor and Uhlig model). Write the equations for this model and discuss each of them. Then discuss how the model can be represented in Excel or GAMS. Then discuss the kinds of results that are obtained when the model is solved. Comment briefly on whether you would prefer to setup and solve this type of model in GAMS or in Excel and explain why.

(b) Discuss the Hall and Taylor model and its implementation in GAMS. Begin with a rough outline of the equations. Discuss in outlines the methods use to (1) solve the steady state model, (2) solve the dynamic nonlinear model and (3) the steps to linearize the model and solve the linearized version.

Question 2. Please answer *either* part (a) or part (b).

(a) A variety of programming languages and software systems are used in computational economics. Consider three of these such as GAMS, GAUSS, Mathematica, Duali, Access, Fortran and C. Briefly discuss their structure, their strengths and weaknesses. Then describe what kind of economic models you would use each of the types of software to develop.

(b) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

Part B

Complete this if you took Wilcoxen's CGE course to finish the field.

Please answer *each* of the questions in this section.

(1) Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

(2) What issues are important in designing the production sector of a general equilibrium model? Be sure your discussion covers at least the following: data sources, the merits of cost versus production functions and choice of functional forms.

The remaining questions in Part B involve the following model. Consider an economy in which the intertemporal behavior of households can be captured by a utility function of the following form:

$$U = \int_0^\infty \ln(u) e^{-0.05s}\, ds$$

where u is the instantaneous utility, or felicity, received by the households at each point in time. It is a function of the households' consumption of goods C and leisure J as shown below:

$$u = C^{0.5} J^{0.5}$$

Households have an endowment of time, H, which they may allocate between leisure and labor, L. There is a single firm producing a good Q from inputs of labor and capital, K. The firm has a Leontief production function of the form:

$$Q = \min\{\frac{L}{2}, K\}$$

The good can be consumed or used for investment. The government collects and consumes T units of the good as a tax. The capital stock evolves according to the following accumulation equation:

$$\frac{dK}{dt} = I - 0.2K$$

where I is the quantity of the good used for investment.

Please answer the following questions.

(3) Derive the equations of motion for the model and find the steady state in (K,λ) space.

(4) Determine the stability of the model near the steady state.

(5) Construct a phase diagram for this model in (K,λ) space, where λ is the costate variable. Be sure to show the steady state, the isoclines, the directions of motion and the stable path. Show all your work and label everything appropriately.

(6) Now make a copy of the diagram and use it to analyze a surprise temporary increase in T lasting for several years. In addition, draw appropriate integral curves for λ, K, J, Q and C. You may assume that the model is initially at its steady state.

Part C

Complete this if you took Kendrick's Stochastic Control course to finish the field.

Please answer any *two* of the following three questions.

(1) Consider the nonlinear active learning stochastic control problem discussed in Chapter 9 of the "Stochastic Control for Economic Models" book. Outline the problem statement including the criterion function, the systems equations, the measurement equations and the stochastic elements. The solution to this model is developed later in this chapter in terms of an expression for the optimal cost-to-go, $J^*_{d,N-k}$. There are two versions of this expression. Discuss the first of these two and, in anticipation of the results in Chapter 10, describe the three components of the cost-to-go.

(2) Discuss the updating rules which are used in Chapter 10 for the original state vector, x, and the parameter vector, theta, for both the means and the covariances. Explain the differences between the (k|k), (k+1|k) and (k+1|k+1) terms for both the means and the covariances for both the original state, x, and the parameter vector, theta.

(3) The Duali software can be used to solve both certainty equivalence (CE) models with additive noise and open loop feedback (OLF) stochastic control problems with both additive and multiplicative noise terms. Describe the stochastic parts of the input for these models, i.e. both additive and multiplicative stochastic elements and discuss how these data are input to Duali. Discuss the nature of the output when the solutions to CE and OLF models are solved and the solutions compared.

Department of Economics
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Comprehensive Exam
Computational Economics
August 2002

Part A

Question 1. Please answer *either* part (a) or part (b).

(a) Discuss the use of the Access software to represent the relational database of the U.S. economy. First discuss the relational database itself. What are its components and how are they related? Then discuss how the database is represented in Access and how the software is used to develop relational queries.

(b) Discuss the Hall and Taylor model and its implementation in GAMS. Begin with a rough outline of the equations. Discuss in outlines the methods use to (1) solve the steady state model, (2) solve the dynamic nonlinear model and (3) the steps to linearize the model and solve the linearized version.

Question 2. Please answer *either* part (a) or part (b).

(a) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

(b) Discuss the use of neural nets for estimation in economic models. Begin with the mathematics of the subject, continue with the way this is implemented in computer software and then discuss a potential application to an economic model. Finally, discuss the pitfalls of using neural nets and compare the use of neural nets to more conventional methods of estimation.

Part B

Please answer any *two* (2) of the following three questions.

(3) Consider the quadratic-linear active learning stochastic control problem discusses in Ch. 10 of the "Stochastic Control for Economic Models" book. Provide a mathematical description of the problem statement, paying particular attention to the various sources of uncertainty. Briefly explain how the model is set up for solution by augmenting the state vector. Describe the three components of the cost-to-go and their approximate shapes as a function of the search value of the control.

(4) Discuss the updating rules which are used in Ch. 10 for the original state vector, x, and the parameter vector, theta, for both the means and the covariances. Explain the differences between the (k|k), (k+1|k) and (k+1|k+1) terms for both the means and the covariances for both the original state, x, and the parameter vector, theta.

(5) The Duali software can be used to solve both certainty equivalence (CE) models with additive noise and open loop feedback (OLF) stochastic control problems with both additive and multiplicative noise terms. Describe the stochastic parts of the input for these models, i.e. both additive and multiplicative stochastic elements and discuss how these data are input to Duali. Discuss the nature of the output when the solutions to CE and OLF models are solved and the solutions compared.

Department of Economics
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Comprehensive Exam
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January 2002

This exam is divided into three parts. Part A is required for everyone. Parts B and C are alternatives: Part B is required if you took Prof. Wilcoxen's course on CGE models and Part C is required if you took the Stochastic Control for Economic Models reading course from Prof. Kendrick as a substitute for Prof. Wilcoxen's course.

Part A

Everyone should complete this section.

Question 1. Please answer *either* part (a) or part (b).

(a) Consider the Chakravarty single sector growth model (or its more recent form as the Taylor and Uhlig model). Write the equations for this model and discuss each of them. Then discuss how the model can be represented in Excel or GAMS. Then discuss the kinds of results that are obtained when the model is solved. Comment briefly on whether you would prefer to setup and solve this type of model in GAMS or in Excel and explain why.

(b) Discuss the Hall and Taylor model and its implementation in GAMS. Begin with a rough outline of the equations. Discuss in outlines the methods use to (1) solve the steady state model, (2) solve the dynamic nonlinear model and (3) the steps to linearize the model and solve the linearized version.

Question 2. Please answer *either* part (a) or part (b).

(a) Compare the GAUSS and GAMS programming languages. Begin by discussing some of the structure of the two languages, i.e. the key words, the organization of elements of a model, the use of sets, the input of data, the specification of the equation or inequalities and the criterion function, the solution of the model, and the display of the results. Then discuss the comparative advantage of each language. What kinds of economic models are most easily and efficiently represented and solved in each language?

(b) Discuss the use of neural nets for estimation in economic models. Begin with the mathematics of the subject, continue with the way this is implemented in computer software and then discuss a potential application to an economic model. Finally, discuss the pitfalls of using neural nets and compare the use of neural nets to more conventional methods of estimation.

Part B

Complete this if you took Wilcoxen's CGE course to finish the field.

Please answer *each* of the questions in this section.

(1) Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

(2) What is input-output data? Why and how is it used in general equilibrium models? Please discuss in detail.

(3) Discuss several ways in which saving has been represented in general equilibrium models. Evaluate the strengths and weaknesses of each approach.

The remaining questions in Part B involve the following model. Consider an economy in which the intertemporal behavior of households can be captured by a utility function of the following form:

$$U = \int_0^\infty \ln(u) e^{-0.05s} ds$$

where u is the instantaneous utility, or felicity, received by the households at each point in time. It is a function of the households' consumption of goods x and y, C_x and C_y, as shown below:

$$u = C_x^{0.5} C_y^{0.5}$$

Households have an endowment of labor, L, which they supply inelastically. Output of x and y is linear in labor and capital, respectively: $Q_x = L$ and $Q_y = K$. In addition to being consumed, good x (the good produced using labor) is used for investment. The government collects and consumes T units of good y as a tax. The capital stock evolves according to the following accumulation equation:

$$\frac{dK}{dt} = I - 0.1K$$

where I is the quantity of good x used for investment.

Please answer the following questions.

(4) Derive the equations of motion for the model and find the steady state.

(5) Determine the stability of the model near the steady state.

(6) Construct a phase diagram for this model in (λ,K) space, where λ is the costate variable. Be sure to show the steady state, the isoclines, the directions of motion and the stable path. Show all your work and label everything appropriately.

(7) Now make a copy of the diagram and use it to analyze a temporary increase in T taking place immediately and lasting for four years. In addition, draw appropriate integral curves for λ, K, I, C_x and C_y. Explain why the C_x and C_y trajectories look the way they do.

Part C

Complete this if you took Kendrick's Stochastic Control course to finish the field.

Please answer any *two* of the following three questions.

(1) Consider the quadratic-linear active learning stochastic control problem discusses in Chapter 10 of the "Stochastic Control for Economic Models" book. Provide a mathematical description of the problem statement, paying particular attention to the various sources of uncertainty. Briefly explain how the model is set up for solution by augmenting the state vector. Describe the three components of the cost-to-go and their approximate shapes as a function of the search value of the control.

(2) Discuss the updating rules which are used in Chapter 10 for the original state vector, x, and the parameter vector, theta, for both the means and the covariances. Explain the differences between the (k|k), (k+1|k) and (k+1|k+1) terms for both the means and the covariances for both the original state, x, and the parameter vector, theta.

(3) The Duali software can be used to solve both certainty equivalence (CE) models with additive noise and open loop feedback (OLF) stochastic control problems with both additive and multiplicative noise terms. Describe the stochastic parts of the input for these models, i.e. both additive and multiplicative stochastic elements and discuss how these data are input to Duali. Discuss the nature of the output when the solutions to CE and OLF models are solved and the solutions compared.

Department of Economics
The University of Texas at Austin

Comprehensive Exam
Computational Economics
August 2001

Part A

Question 1. Please answer *either* part (a) or part (b).

(a) Consider the Chakravarty single sector growth model (or its more recent form as the Taylor and Uhlig model). Write the equations for this model and discuss each of them. Then discuss how the model can be represented in Excel or GAMS. Then discuss briefly the kinds of results that are obtained when the model is solved. Comment briefly on whether you would prefer to setup and solve this type of model in GAMS or in Excel and explain why.

(b) Discuss in outline form the mathematics of how the feedback rule is derived in quadratic-linear optimal control problems and how the algorithm to solve this model is implemented in GAUSS.

Question 2. Please answer *either* part (a) or part (b).

(a) Discuss the use of the Access software to represent the relational database of the US economy. First discuss the relational database itself. What are its components and how are they related? Then discuss how the database is represented in Access and how the software is used to develop relational queries.

(b) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

Part B

Please answer each of the questions in this section.

(3) Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

(4) Discuss the main considerations in the design of the production side of a computational general equilibrium model. Be sure to cover the following items: how you would choose the number of sectors to use; how you would represent their behavior; functional forms; any limitations or considerations imposed by data; considerations relevant to estimation.

(5) How can you tell if a general equilibrium model is working correctly? Please discuss as thoroughly as possible.

Part C

Suppose a firm produces output and also builds its own capital goods. The firm uses no inputs other than capital and its production function is given by $q = \ln(k)$, where q is output and k is the capital stock. Output sells for price p, which the firm takes a given. The firm is subject to convex costs of adjusting its capital stock: buying I units of new capital ends up costing the firm $cI^2/(2\theta)$, where c is the price of raw capital goods (which the firm takes a given) and θ is a parameter. The only tax is on capital gains and is given by variable T. The firm's capital accumulation constraint has the usual form. You may assume that the firm's objective is to maximize its stock market value, V, which can be shown to be given by the following expression:

$$V = \int_0^{\infty} \frac{1}{1-T}\left(p\ln(k) - \frac{cI^2}{2\theta}\right) e^{\left(\frac{-rt}{1-T}\right)} dt$$

(6) Show that the equations of motion in (λ,k) space describing the firm's optimum behavior are the following (where λ is the current-value multiplier):

$$\dot{\lambda} = (\frac{r}{1-T} + \delta)\lambda - \frac{p}{(1-T)k}$$
$$\dot{k} = \frac{\theta\lambda(1-T)}{c} - \delta k$$

(7) Draw an appropriate phase diagram for this firm. Put λ on the vertical axis and k on the horizontal axis. Be sure to label everything and *show your work*. Do not just sketch a general phase diagram– derive the one appropriate for this problem.

(8) Show that the model is saddle-path stable near the steady state. Why is saddle-path stability important? Discuss.

(9) Copy the diagram from (7) and use it to analyze a *surprise, temporary reduction* in p. The drop in p takes effect immediately but is known to last only 2 years, after which p will return to its original value. Show the trajectory in the phase diagram and draw appropriate integral curves. You may assume that the system is initially at the steady state. Explain your results in words.

Department of Economics
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Comprehensive Exam
Computational Economics
January 2001

Part A

*Answer any **2** of the following **3** questions.*
*This section as a whole will count for **50%** of your grade.*

1. A variety of programming languages and software systems are used in computational economics. Consider three of these such as GAMS, GAUSS, Mathematica, Duali, Access, Fortran and C. Briefly discuss their structure, their strengths and weaknesses. Then describe what kind of economic models you would use each of the types of software to develop.
2. Discuss the use of neural nets for estimation in economic models. Begin with the mathematics of the subject, continue with the way this is implemented in computer software and then discuss a potential application to an economic model. Finally, discuss the pitfalls of using neural nets and compare the use of neural nets to more conventional methods of estimation.
3. Discuss the Hall and Taylor model and its implementation in GAMS. Begin with a rough outline of the equations. Discuss in outlines the methods use to (1) solve the steady state model, (2) solve the dynamic nonlinear model and (3) the steps to linearize the model and solve the linearized version.

Part B

*Answer any **2** of the following **4** questions.*
*This section as a whole will count for **20%** of your grade.*

1. Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

2. What is the principal source of data used to parameterize the production side of a multi-sector general equilibrium model? Discuss the availability of this data and how that influences the choice of functional forms one might use for modeling production.

3. What additional features are needed when modeling open economies? Briefly explain how one might deal with each of the these in the model.

4. How might one incorporate technical change into a general equilibrium model? Given the historical record, what features would the technical change specification need to have? What problems would arise if one were trying to build a model with both technical change and increasing returns to scale?

Part C

*Answer **all** of the remaining questions.*
*This section as a whole will count for **30%** of your grade.*

Many empirical studies have shown that firms face adjustment costs when changing the amount of labor they use. In fact, for many firms it can be harder to adjust the labor force than to change the capital stock. For example, many firms in the service sector employ highly skilled people but use comparatively small amounts of general-purpose capital. This problem explores that issue.

A price-taking firm's output, Q, is a Cobb-Douglas function of labor (L) and intermediate goods (X): $Q = L^{\alpha}X^{1-\alpha}$, where α is a parameter. It pays w per unit of L and receives a price p per unit of output. In addition, it is costly for the firm to change its number of employees (that is, there are costs of hiring and firing workers). As a result, the firm's labor input is fixed in the short run and evolves over time according to the following accumulation equation:

$$\dot{L} = h - \delta L$$

where h is the rate of new hires and δ is an exogenous rate at which current employees leave the firm.

1. Show that the firm's short run profit, π, on a given stock of labor can be written in the form: $\pi=\beta(p,w)L$, where β is a function of p and w.

Over the long run, the firm wishes to choose its hiring so as to maximize the stock market value of its equity, which is given by the following:

$$V = \int (1-T)\big(\beta L - C(h)\big)e^{-rt}\,dt$$

where T is the corporate profits tax, $C(h)$ is the cost of hiring h new workers (the search and training cost, *not* their wages), and r is the interest rate. To keep things simple, let $C(h)$ be wh^2 (think of it as the cost of operating the personnel office).

2. Derive the equations of motion for this model in terms of the costate variable (call it λ) and L and solve for the steady state.

3. Show that the model is saddle-path stable near the steady state. Why is this important?

4. Choose plausible values for any key parameters and use them to discuss the speed at which employment will respond to a shock. Be as quantitative as possible.

5. Construct a phase diagram for the model. Show the steady state, the isoclines, the directions of motion and the stable path. Please put λ on the vertical axis and be sure to label everything.

Suppose the government wants to raise employment by cutting T. The cut is announced today but will not take effect for two years. Once T has been cut, however, the reduction will be permanent.

6. Make a copy of the diagram from (5) and use it to analyze the policy. Show the trajectory in the phase diagram and draw appropriate integral curves. You may assume that the system is initially at

the steady state from part (5). Referring back to part (4), discuss whether tax cuts are a reasonable tool for dealing with short run employment problems. Explain your reasoning.

Department of Economics
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Comprehensive Exam
Computational Economics
August 2000

Part A

Question 1. Please answer *either* part (a) or part (b).

(a) Consider the Chakravarty single sector growth model. Write the equations for this model and discuss each of them. Then discuss how the model can be represented in GAMS. Then discuss briefly the kinds of results that are obtained when the model is solved. Comment briefly on whether you would prefer to setup and solve this type of model in GAMS or in Excel and explain why.

(b) Discuss the use of the Access software to represent the relational database of the US economy. First discuss the relational database itself. What are its components and how are they related? Then discuss how the database is represented in Access and how the software is used to develop relational queries.

Question 2. Please answer *either* part (a) or part (b).

(a) Discuss the Hall and Taylor model and its implementation in GAMS. Begin with a rough outline of the equations. Discuss in outlines the methods used to (1) solve the steady state model, (2) solve the dynamic nonlinear model, and (3) the steps to linearize the model and solve the linearized version.

(b) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

Part B

Please answer *all* of the following questions.

(1) Discuss the design of the production side of a general equilibrium model. Be sure to address at least the following: data sources, choice of functional forms, cost versus production functions and technical change.

(2) What numerical methods are used to solve intertemporal general equilibrium models? Explain how each one works and what its strengths and weaknesses are.

Part B, continued.

For the remaining questions, consider an economy in which the intertemporal behavior of households can be captured by a utility function of the following form:

$$U = \int_0^{\infty} \ln(C - \gamma D) e^{-\rho s} ds$$

where ρ is the rate of time preference, C is the quantity of goods consumed at a particular instant of time, D is the level of pollution (which people dislike) and γ is a parameter. Households also supply labor, L, and their labor supply is perfectly inelastic. The economy's gross output at any particular time is given by a production function of the form:

$$Q = L^{0.5} N^{0.5}$$

where N is a Leontief function of the capital stock, K, and the level of pollution:

$$N = \min\{K, D\}$$

The government consumes an exogenous amount G of the gross output and the remainder can be divided between consumption, C, and investment, I. The capital stock evolves according to an accumulation equation of the usual form.

(3) Derive the equations of motion for this model in terms of the costate variable (call it λ) and K and solve for the steady state values of K, C, D, and λ.

(4) How would the steady state respond to an increase in G? Explain the intuition behind your results.

(5) How would the original steady state (from part 3) respond to an increase in γ? Discuss.

(6) Explain how you would show that the model is saddle-path stable near the steady state and then discuss why this property is important. (Note: please explain how you would do this, don't actually do it.)

(7) Construct a phase diagram for the model. Show the steady state, the isoclines, the directions of motion and the stable path. Please put λ on the vertical axis and be sure to label everything.

(8) Make a copy of the diagram from (7) and use it to analyze a sudden, permanent increase in γ. Show the trajectory in the phase diagram and draw appropriate integral curves. You may assume that the system is initially at the steady state from part (3). Explain in words what is happens.

Department of Economics
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Comprehensive Exam
Computational Economics
August 1999

Part A

There are three questions in this section. *Please answer **any two** of them*.

Question 1

Please answer *either* part (a) or part (b):

(a) A variety of programming languages and software systems are used in computational economics. Consider three of these such as GAMS, GAUSS, Mathematica, DUALI, Access, Fortran and C. Briefly discuss their structure, their strengths and weaknesses. Then describe what kind of economic models you would use each of the types of software to develop.

(b) Discuss in outline form the mathematics of how the feedback rule is derived in quadratic-linear optimal control problems and how the algorithm to solve this model is implemented in GAUSS.

Question 2

Please answer *either* part (a), part (b) or part (c):

(a) Discuss the Hall and Taylor model and its implementation in GAMS. Begin with a rough outline of the equations. Discuss in outlines the methods use to (1) solve the steady state model, (2) solve the dynamic nonlinear model and (3) the steps to linearize the model and solve the linearized version.

(b) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

Part A, continued.

Question 3

Please answer *any two* of parts (a), (b) and (c):

(a) What is the fundamental difference between calibration and estimation? Discuss. Please go beyond a superficial description of how the two are carried out.

(b) Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

(c) The representative agent approach is commonly used to model household behavior in general equilibrium models. Discuss the strengths and weaknesses of this approach as thoroughly as you can.

Part B

Please answer *all* of Question 4.

Question 4

Consider an economy in which the intertemporal behavior of households can be captured by a utility function of the following form:

$$U = \int_0^{\infty} \ln c(s) e^{-\rho s} ds$$

where $c(s)$ is the quantity of goods consumed at time s and ρ is the rate of time preference. The economy's gross output at time s, $q(s)$, is given by a production function of the form:

$$q(s) = k(s)^{\alpha}$$

where k(s) is the capital stock and α is a parameter strictly less than one. The government consumes an exogenous amount T of the gross output and the remainder can be divided between consumption, c, and investment, I.

(a) Derive the equations of motion for this model in terms of the costate variable (call it λ) and k and solve for the steady state.

(b) Show that the model is saddle-path stable near the steady state. Why is this important?

(c) Construct a phase diagram for the model. Show the steady state, the isoclines, the directions of motion and the stable path. Please put λ on the vertical axis and be sure to label everything.

Now suppose the government announces a temporary increase in the tax T. The increase is announced today, takes effect immediately, and will last for three years. After that, the tax will go back to its original level.

(d) Make a copy of the diagram from (c) and use it to analyze the policy. Show the trajectory in the phase diagram and draw appropriate integral curves. You may assume that the system is initially at the steady state from part (c). Explain in words what is happens.

Department of Economics
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Comprehensive Exam
Computational Economics
January 1999

Part A

There are three questions in this section. *Please answer any **two** of them.*

Question 1

Please answer *either* part (a) or part (b):

(a) A variety of programming languages and software systems are used in computational economics. Consider three of these such as GAMS, GAUSS, Mathematica, DUALI, Access, Fortran and C. Briefly Discuss their structure, their strengths and weaknesses. Then describe what kind of economic models you would use each of the types of software to develop.

(b) Consider the Taylor and Uhlig single sector growth model. Write the equations for this model and discuss each of them. Then discuss how the model can be represented in Excel. Then describe how the Solver is used to optimize this model in Excel. Finally, comment on whether you would prefer to setup and solve this model in Excel or in GAMS and explain why.

Question 2

Please answer *either* part (a), part (b) or part (c):

(a) Relational databases offer a useful way to organize economic data. Describe the theory of relational databases – in particular the use of joins. Then discuss the implementation of a relation database in some software system such as Access. Finally, use an economic example to illustrate how a relational database might be implement in economics.

(b) Consider the Hall and Taylor in GAMS. Describe the basic equations of the model such as consumption, investment, money demand, prices and exchange rates. Then discuss the method of representing the model in GAMS and solving it for (1) the nonlinear steady state solution and (2) the nonlinear dynamic solution.

(c) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

Part A, continued.

Question 3

Please answer *any two* of parts (a), (b) and (c):

(a) Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

(b) Discuss several ways in which savings could be represented in a general equilibrium model. Explain when each would be appropriate.

(c) Ideally, one might like to use a fully flexible functional form to represent production in each industry of a multisector model. Why might this not be possible? How would one cope with the problem?

Part B

Please answer *all* of Question 4.

Question 4

A price-taking firm's output, q, is a Cobb-Douglas function of its inputs of labor and capital: $q = K^{\alpha}L^{1-\alpha}$, where α is a parameter. It pays w per unit of labor and receives a price p per unit of output.

(a) Show that the firm's short run profit, π, on its capital stock can be written in the form: $\pi=\beta(p,w)K$, where β is a function of p and w.

Over the long run, the firm wishes to choose its investment so as to maximize the stock market value of its equity, which is given by the following:

$$V = \int (1-T)\left(\beta K - P_k I\right)e^{-rt}\,dt$$

where T is the corporate profits tax, P_k is the price of new capital goods, I is the quantity of investment and r is the interest rate. The firm's capital accumulation is subject to the constraint:

$$\dot{K} = I^{\phi} - \delta K$$

where ϕ is a parameter capturing the effect of adjustment costs and is strictly less than 1. The depreciation rate is δ.

(b) Derive the equations of motion for this model in terms of the costate variable (call it λ) and K and solve for the steady state.

(c) Show that the model is saddle-path stable near the steady state. Why is this important?

(d) Construct a phase diagram for the model. Show the steady state, the isoclines, the directions of motion and the stable path. Please put λ on the vertical axis and be sure to label everything.

Suppose Congress wants to expand the economy by cutting T. The cut is announced today but will not take effect for two years. Once T has been cut, however, the reduction will be permanent.

(e) Make a copy of the diagram from (d) and use it to analyze the policy. Show the trajectory in the phase diagram and draw appropriate integral curves. You may assume that the system is initially at the steady state from part (d). Finally, evaluate the policy.

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Comprehensive Exam
Computational Economics
August 1998

This exam has two sections, A and B. Please do **any two** *of the three questions in part A and* **all** *of part B.*

Section A

There are three questions in this section: A1, A2 and A3. *Please answer any* ***two*** *of them*.

Question A1

Please answer *either* part (a) or part (b):

(a) Discuss the use of the Access software to represent the relational database of the U.S. economy. First discuss the relational database itself. What are its components and how are they related? Then discuss how the database is represented in Access and how the software is used to develop relational queries.

(b) Discuss the use of neural nets for estimation in economic models. Begin with the mathematics of the subject, continue with the way this is implemented in computer software and then discuss a potential application to an economic model. Finally, discuss the pitfalls of using neural nets and compare the use f neural nets to more conventional methods of estimation.

Question A2

Please answer *either* part (a) or part (b):

(a) Compare the GAUSS and GAMS programming languages. Begin by discussing some of the structure of the two languages, i.e. the key words, the organization of elements of a model, the use of sets, the input of data, the specification of the equation or inequalities and the criterion function, the solution of the model, and the display of the results. Then discuss the comparative advantage of each language. What kinds of economic models are most easily and efficiently represented and solved in each language?

(b) Consider either the Miniature Orani model of Dixon, et al., or the CGE model a la Bruce Smith. Describe briefly the key elements of the mathematics of the model, then how this is implemented in the GAMS modeling language. Finally, discuss the limitations of the model and what you might do to improve it.

Question A3

Please answer *any two* of parts (a), (b) and (c):

(a) Discuss at least three ways labor supply could be represented in a general equilibrium model. Explain when each would be appropriate.

(b) Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

(c) How do you know if a general equilibrium model is working correctly? Discuss in detail.

Please be sure to answer the questions in Section B on the next page.

Section B

There is one question in this section. Please answer *all* parts of it.

Question B1

A price-taking firm wishes to maximize the stock market value of its equity, which is given by the following:

$$V = \int \left(p(1-T)q - wL \right) e^{-rt} dt$$

where p is the purchaser's price of the firm's output, T is a tax rate, q is the firm's output, w is the wage rate, L is labor and r is the interest rate. The firm uses only labor and is subject to the production function:

$$q = L^{\alpha} - h^2$$

where α is a parameter less than 1 and h is the rate of change of the firm's labor force:

$$\frac{dL}{dt} = h$$

(a) Derive the equations of motion for this model in terms of L and h and solve for the steady state.

(b) Show that the model is saddle-path stable near the steady state. Why is this important?

(c) Construct a phase diagram for the model. Show the steady state, the isoclines, the directions of motion and the stable path. Please put h on the vertical axis and be sure to label everything.

(d) Now make a copy of the diagram and use it to analyze an anticipated cut in T two years in the future. Show the trajectory in the phase diagram and draw integral curves for h and L. You may assume that the system is initially at the steady state from part (4). Explain your results in words.

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January 1998

Please answer ALL of the questions in Section A and TWO of the three questions in Section B.

Section A

(1) Discuss the design of the production portion of a multisector general equilibrium model. Be sure to cover at least the following: data sources; considerations in the choice of functional form; parameterization; economies of scale; and technical change.

(2) Discuss several numerical methods that could be used to solve static or intertemporal general equilibrium models. Explain how each one works and what its strengths and weaknesses are.

(3) How do you know if a general equilibrium model is working correctly? Discuss in detail.

Please use the following information to answer questions 4 through 7. A price-taking firm wishes to maximize the stock market value of its equity, which is given by the following:

$$V = \int (p(1-T)q - c(1-S)I^{\phi})e^{-rt}dt$$

subject to the production function and accumulation constraint shown below:

$$q = k$$

$$\frac{dk}{dt} = I - \delta k$$

where p is the purchaser's price of the firm's output, T is a tax rate, k is the firm's capital stock, c is the price of investment goods, S is an investment subsidy, I is the amount of investment, r is the interest rate, and ϕ and δ are parameters. You may assume that $\phi > 1$.

(4) Show that the optimum for this problem can be characterized by the following equations of motion:

$$\frac{d\lambda}{dt} = (r+\delta)\lambda - p(1-T)$$

$$\frac{dk}{dt} = \left(\frac{\lambda}{c(1-S)\phi}\right)^{1/(\phi-1)} - \delta k$$

where λ is the multiplier associated with the accumulation constraint.

(5) Show that the model is saddle-path stable near the steady state. Why is this important?

(6) Construct a phase diagram for this model. Be sure to show the steady state, the isoclines, the directions of motion and the stable path. Also, be sure to put the costate variable on the vertical axis. Finally, show all your work.

(7) Now make a copy of the diagram and use it to analyze two experiments: a surprise increase in *T* taking place immediately, and an anticipated increase in *T* two years in the future. Draw a separate phase diagram for each experiment and draw appropriate integral curves for λ, *I* and *k*.

Section B

1. Consider the neural net in an Excel spreadsheet developed by Hans Amman on the basis of a chapter from Tom Sargent's book. First describe this model mathematically. Then describe how it is implemented in the Excel spreadsheet. Finally, discuss the limitations of this kind of neural net methods for estimation.

2. Describe how the GAUSS language is used to model and solve the quadratic-linear dynamic optimal control model. Use a rough idea of the mathematics and then describe how this is represented in GAUSS statements. Also, provide a critique of the GAUSS language in this use.

3. Answer either part (a) or part (b):

   (a) Consider either the Miniature Orani model of Dixon, et. al or the CGE model a la Bruce Smith. Describe briefly the key elements of the mathematics of the model, then how this is implemented in the GAMS modeling language. Finally, discuss the limitations of the model and what you might do to improve it.

   (b) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

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Section A

There are seven questions in this section. *Please answer **all** of them.*

(1) What is the fundamental difference between calibration and estimation? Discuss. Please go beyond a superficial description of how the two are carried out.

(2) What issues are important in designing the production sector of a general equilibrium model? Be sure your discussion covers at least the following: data sources, the merits of cost versus production functions, functional forms and technical change.

(3) How might international capital flows be modeled in a multi-country general equilibrium model? What features are needed? Why is it important to model capital flows?

Please use the following information to answer questions 4 through 7.

Consider an economy composed of a representative household and a single firm. The household's intertemporal utility function takes the following form:

$$u = \int \ln(C - \bar{c})\, e^{-\rho s} ds$$

where ρ is the time preference rate, $\bar{c}$ is a parameter, and C is the household's consumption of goods. The household has an exogenous endowment of labor, L, and also owns the capital stock. The household buys investment goods in order to increase the capital stock and is subject to the following accumulation condition:

$$\frac{dK}{dt} = I - \delta K$$

The firm makes output, X, out of labor and capital according to the following production function:

$$X = \beta L^{\alpha} K^{1-\alpha}$$

Output can be used for either consumption or investment.

(4) Set up the Ramsey-style optimization problem for this economy. Derive the equations of motion in (C,K) space and find expressions for the steady state.

(5) Show that the model from (a) is saddle-path stable near the steady state. Why is this important?

(6) Construct an appropriate phase diagram for this model and label everything clearly. (Do *not* just draw a generic phase diagram -- it must be the appropriate one for this problem.)

(7) Now make a copy of the diagram and use it to analyze an anticipated permanent increase in the level of technology, β. Show the change in the steady state, the trajectory of the model in the phase diagram, and the integral curves for C and K.

Section B
Please answer two of the following three questions.

1. Discuss the use of neural nets for estimation in economic models. Begin with the mathematics of the subject, continue with the way this is implemented in computer software and then discuss a potential application to an economic model. Finally, discuss the pitfalls of using neural nets and compare the use of neural nets to more conventional methods of estimation.

2. Relational databases offer a useful way to organize economic data. Describe the theory of relational databases – in particular, the use of joins. Then discuss the implementation of a relational database in some software system such as Access. Finally, use an economic example to illustrate how a relational database might be implemented in economics.

3. Compare the GAUSS and GAMS programming languages. Begin by discussing some of the structure of the two languages, i.e. the key words, the organization of elements of a model, the use of sets, the input of data, the specification of the equations or inequalities and the criterion function, the solution of the model, and the display of the results. Then discuss the comparative advantage of each language. What kinds of economic models are most easily and efficiently represented and solved in each language?

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Comprehensive Exam
Computational Economics
January 1997

This exam has two sections, A and B.
*You must complete any **two** of the three questions in Section A and*
***all eight** of the questions in Section B.*

Section A

Please answer any two of the following three questions.

(1) The GAMS modeling language can be used to develop general equilibrium models of economies. Discuss the mathematical specification of a simple general equilibrium model and then describe how this specification is input to GAMS. Then describe an experiment on the model and discuss the types of results produced by the GAMS system and your interpretation of those results for the experiment you have proposed. You may want to draw on the Canadian model of Bruce Smith to answer this question.

(2) Answer either (a) or (b) below:

(a) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Then critique the model and tell how you would improve it.

(b) Discuss the use of regression trees and neural nets for estimating economic models. Outline the methods and then discuss their strengths and weaknesses.

(3) A variety of programming languages and software systems are used in computational economics. Consider three of these such as GAMS, GAUSS, Mathematica, DUALI, Access, Fortran and C. Briefly discuss their structure and their strengths and weaknesses. Then describe what kind of economic models you would use each of the types of software to develop.

Section B

There are eight questions in this section. *Please answer **all** of them.*

(1) How might one incorporate technical change into a general equilibrium model? Given the historical record, what features would the technical change specification need to have? What problems would arise if one were trying to build a model with both technical change and increasing returns to scale?

(2) What additional features are needed when modeling open economies? Briefly explain how one might deal with each of the these in the model.

(3) What is the principal source of data used to parameterize the production side of a multi-sector general equilibrium model? Discuss the availability of this data and how that influences the choice of functional forms one might use for modeling production.

Please use the following information to answer questions 4 through 8.

Consider an economy composed of a representative household and a single firm. The household's intertemporal utility function takes the following form:

$$u = \int \ln(C)\, e^{-\rho s} ds$$

where ρ is the time preference rate and C is a Cobb-Douglas function of leisure (J) and the household's consumption of goods (G):

$$C = J^{1/2} G^{1/2}$$

The household has an endowment of time, N, which can be allocated to labor (L) or leisure subject to the usual constraint:

$$L + J = N$$

When the household works it accumulates experience which leads to the formation of human capital (H) according to the equation:

$$\frac{dH}{dt} = L - \delta H$$

Human capital depreciates at rate δ. Finally, the firm makes consumption goods (G) out of human capital according to the production function:

$$G = H^{1/2}$$

(4) Set up the Ramsey-style optimization problem for this economy. Derive the equations of motion in (H,L) space and find expressions for the steady state.

(5) Show that the model from (a) is saddle-path stable near the steady state. Why is this important?

(6) Construct an appropriate phase diagram for this model and label everything clearly. (Do *not* just draw a generic phase diagram -- it must be the appropriate one for this problem.)

(7) Now make a copy of the diagram and use it to analyze an anticipated permanent increase in the endowment of hours, N. Show the change in the steady state, the trajectory of the model in the phase diagram, and the integral curves for L and H.

(8) What is happening to leisure along this trajectory? Explain intuitively why this happens.

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Section A
Please answer both parts of this section.

Part 1

Consider a world made up of two economies. Each economy consists of three agents: an infinitely-lived representative household, a single firm and a government. Let the good produced by the firm in country 1 be called *X* and the good produced in country 2 be *Y*. Suppose the households in the two countries maximize utility functions of the form:

$$U_i = \int_0^\infty \ln(C_i) e^{-\rho s} ds$$

where ρ is the same in both countries and C_i is an aggregate consumption variable defined as follows:

$$C_i = X_i^{1/2} Y_i^{1/2}$$

and X_i and Y_i are the amounts of *X* and *Y* consumed by households in country *i*. In each economy, production uses only capital:

$$X^s = K_1^{1/2}, \quad Y^s = K_2^{1/2}$$

where K_i is the capital stock in country *i* and X^s and Y^s are the total supplies of goods *X* and *Y*. The capital stocks evolve according to the usual accumulation conditions:

$$\dot{K}_i = I_i - \delta K_i$$

where I_i is the amount of country *i*'s domestic good used for investment (in country 1 this would be the *X* good, for example), and δ is the same in the two countries. The government in country *i* collects a tax T_i of the goods produced in that country (this is the total tax in goods, not a rate). Finally, the counties will trade with each other subject to the constraint:

$$X_2 = \phi Y_1$$

where ϕ is the exchange rate of goods between the two countries (the price of Y over the price of X). Assuming that each country is a price-taker in the world market (each takes ϕ as given), please answer the following questions.

(a) Set up the Ramsey-style optimization problem for the household in country 1 and take appropriate first-order conditions. (That is, set up and solve the household's optimization problem taking into account the production function, etc.)

(b) Construct an appropriate phase diagram for country 1 in (C,K) space (still taking ϕ as given). Put C on the vertical axis. Label everything and show your work.

(c) Solve for the model's steady state, including the exchange rate.

(d) Now analyze the steady state effects of the following experiment: an increase in the tax rate in country 2. Be sure to determine the effects (if any) of the experiment on *both* countries. Discuss.

(e) Suppose one wished to implement the full intertemporal version of this model on a computer (not just the steady state). Discuss solution methods that would be appropriate. What are the advantages and disadvantages of each?

Part 2

Discuss the main considerations in the design of the production side of a computational general equilibrium model. Be sure to discuss at least the following items: how many sectors you would use; how you would represent their behavior; functional forms; any limitations or considerations imposed by data; considerations relevant to estimation.

Section B

Please answer two of the following questions.

1. Discuss the use of the Access software to represent the relational database of the U.S. economy. First discuss the relational database itself. What are its components and how are they related? Then discuss how the database is represented in Access and how the software is used to develop relational queries.

2. Describe how the GAUSS language is used to model and solve the quadratic-linear dynamic optimal control model. Use a rough idea of the mathematics and then describe how this is represented in GAUSS statements. Also, provide a critique of the GAUSS language in this use.

3. Answer either part (a) or part (b):

   (a) Discuss the treatment of land, labor and water constraints in agricultural models. Also discuss the use of mean-variance analysis to treat uncertainty in agricultural models. Then critique the existing agricultural models and discuss how you would improve them.

   (b) Discuss the global warming model of Nordhaus. Describe the model in both mathematics and English and then discuss the results. Lastly, critique the model and tell how you would improve it.

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The University of Texas at Austin

Comprehensive Exam
Computational Economics
January 1996

This exam has two sections, A and B.
*You must complete any **two** of the three questions in Section A and*
***both** of the questions in Section B.*

Section A (Answer any 2 of the 3 questions.)

Question 1:

Consider the static and dynamic models discussed in the chapter on sectoral models from Kendrick's "Models for Analyzing Comparative Advantage" book. From the static model discuss the materials balance constraints for raw materials, intermediate products and final products. From the dynamic model discuss the treatment of investment. In each case critique the formulations which are used and describe how you would improve the models.

Question 2

Describe how the GAUSS language is used to model and solve the quadratic-linear dynamic control model. Use a rough idea of the mathematics and then describe how this is represented in GAUSS statements. Also, provide a critique of the GAUSS language in this use.

Question 3

Discuss the use of the Access software to represent the relational database of the U.S. economy. First discuss the relational database itself. What are its components and how are they related? Then discuss how the database is represented in Access and how the software is used to develop relational queries.

Questions 4 and 5 are on the next page.

Section B (Answer BOTH of the questions (4 and 5) in this section.)

Question 4:

Suppose a firm produces output and also builds its own capital goods. The firm uses no inputs other than capital and its production function is given by $q = \ln(k)$, where q is output and k is the capital stock. Output sells for price p, which the firm takes a given. The firm is subject to convex costs of adjusting its capital stock: buying I units of new capital ends up cost the firm $cI^2/(2\theta)$, where c is the price of raw capital goods (which the firm takes a given) and θ is a parameter. The firm's capital accumulation constraint has the usual form. Finally, there is a tax T on capital gains.

(a) Starting from arbitrage conditions, show that if the firm wants to maximize its stock market value its optimization problem would be the following:

$$\max \int_0^\infty \frac{1}{1-T}\left(p\ln(k) - \frac{cI^2}{2\theta}\right)e^{\left(\frac{-r}{1-T}\right)t}dt$$

$$\text{subject to } \dot{k} = I - \delta k$$

(b) Given the problem in part (a), show that the equations of motion in (λ, k) space describing the firm's optimum behavior are the following (where λ is a multiplier):

$$\dot{\lambda} = (r+\delta)\lambda - \frac{p}{(1-T)k}$$

$$\dot{k} = \frac{\theta\lambda(1-T)}{c} - \delta k$$

(c) Draw an appropriate phase diagram for this firm. Put λ on the vertical axis and k on the horizontal axis. Be sure to label everything and *show your work*. Do NOT just sketch a general phase diagram -- derive the one appropriate for this problem.

(d) Copy the diagram from (c) and use it to analyze an *anticipated reduction* in the capital gains tax. Explain your results in words. Be sure to discuss all important details.

(e) Show that the model is saddle-path stable near the steady state. Does the capital gains tax affect the rate at which the model converges to the steady state? Explain.

Question 5

Discuss the principle considerations in designing the household side of a computational general equilibrium model. Be sure to cover at least the following questions: How many households should there be? What sort of decisions do the households make? How should those decisions

be modeled? How would the decision models be parameterized? Where might one get appropriate data.

Comprehensive Exam

GENERAL EQUILIBRIUM AND COMPUTATIONAL ECONOMICS

January 1994

(Please note that this is only the CGE part of the exam.)

Question 3:

Suppose a particular economy can be represented by three agents: a household, a firm and the government. The household has an endowment of hours (H) which it divides between labor (L) and leisure (J). It chooses its consumption of leisure and goods purchased from the firm (X_C) in order to maximize to the following utility function:

$$U = J^{\alpha} X_C^{\alpha}$$

The household's only source of income is the wages it earns supplying labor. It pays taxes to the government equal to T percent of its labor income and spends the remainder of its income on goods. The firm produces a single output and uses only labor as an input. It has the following production function, where X_S is its quantity of output and β is a parameter:

$$X_S = \beta L$$

The government collects the labor tax and spends it buying goods from the firm. It runs a balanced budget at all times. In the remainder of this problem, let w be the wage rate and let p be the price of the firm's output.

(a) Construct a general equilibrium model from the information above. Check that the model is properly closed and discuss which variables are naturally endogenous and which are naturally exogenous.

(b) Assuming that $H=100$, $w=1$ and that $T=0.1$ (10%), solve for the equilibrium in the model above. Calculate all prices and quantities. Explain how the assumption that $w=1$ affects the results.

(c) Explain how you would obtain values of parameters δ and β. Discuss estimation strategies and data sources. Explain the benefits of the method you chose relative to any obvious alternatives.

Question 4:

Suppose a firm uses labor and capital in fixed proportions to produce its output. In particular, suppose it has the following production function:

$$Q = \min\left\{ \frac{K}{a}, \frac{L}{b} \right\}$$

The firm also faces convex internal costs of adjusting its capital stock: it must pay cI^2 dollars for I units of new capital goods, where c is the price of raw capital goods (which the firm takes as given). The firm's objective is to maximize the present value of its dividend stream:

$$V = \int (pQ - wL - cI^2) e^{-rt} dt$$

In this expression, p is the price of the firm's output, w is the wage rate, c is the cost of investment goods, I is the quantity of investment, and r is the interest rate. The firm is also subject to the accumulation constraint:

$$\dot{K} = I - \delta K$$

where δ is the rate of depreciation.

(a) Derive the equations of motion describing this firm's behavior and construct an appropriate phase diagram. Label all important features of the diagram.

(b) Prove that this model has a unique stable path.

(c) Use your phase diagram to analyze how the firm would react to an unanticipated temporary rise in w. That is, imagine that the wage suddenly rises and is expected to remain high for 4 years, after which it is expected to return to its original level.

(d) Discuss several numerical methods that could be used to obtain quantitative solutions to a model of this type. Which method would be best if this model were part of a large general equilibrium model?

Comprehensive Exam

General Equilibrium and Computational Economics

Spring 1993

Answer any three of the following four questions.

Question 1

Describe the treatment of investment and capital stocks in two types of economic models. One of the models should be a sectoral model and the other should be an economy-wide model such as a growth model or a general equilibrium model. Critique the treatment of capital in these models and describe how you would improve the models.

Question 2

Discuss the treatment of land, labor and water constraints in agricultural models. Also discuss the use of mean-variance analysis to treat uncertainty in agricultural models. Then critique the existing agricultural models and discuss how you would improve them.

(... the exam continues on the following pages ...)

Question 3

Suppose a particular economy can be represented by four agents: a household, a firm producing good *X*, a firm producing good *Y*, and the government. The household supplies labor (*L*) and buys goods *X* and *Y*. It also pays a lump sum tax, *T*, to the government. The household's utility function has the following form:

$$U = X^{1/4}Y^{1/4}$$

Firms *X* and *Y* use only labor to produce their outputs and have the following production functions:

$$X = 10L_x$$
$$Y = 40L_y$$

In addition, the government provides an ad valorem subsidy at rate *S* on purchases of good *X*. That is, the price of good *X* to consumers is (1-*S*) times the price received by firm *X*. There is NO subsidy on good *Y* and the government balances its budget at all times.

In the remainder of the problem, let P_X and P_Y be the *producer* prices of *X* and *Y* and let *w* be the wage rate.

(a) Construct a general equilibrium model from the information above. Check that the model is properly closed and discuss which variables are naturally endogenous and which are naturally exogenous.

(b) Assuming that *L*=100, *w*=1 and (for now) that *S*=0, solve for the equilibrium in the model above. Calculate all prices and quantities. Explain how the assumption that *w*=1 affects the results.

(c) Now suppose the government decides to increase the subsidy from zero to 0.4 (that is, the subsidy jumps to 40%). Explain the effects of this change on the economy. Also discuss the effect of the subsidy on an appropriate measure of welfare.

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(... the exam continues on the following page ...)

Question 4

Suppose a particular firm uses only labor to produce its output but faces convex costs of adjusting its labor force. The firm's object is to maximize the stock market value of its equity, which is given by the following:

$$V = \int \left(pq - wL - cH^{\phi} \right) e^{-rt} dt$$

In this expression p and q are the price and quantity of output, w is the wage rate, L is the firm's labor force, H is the rate of new hires, c is the cost of hiring, r is the interest rate, and ϕ is a parameter. The firm is also subject to the production function and accumulation constraint shown below:

$$q = \gamma L$$

$$\frac{dL}{dt} = H - \delta L$$

where γ is a parameter describing the firm's technology and δ is the rate at which existing employees leave the firm.

In the remainder of this problem you may assume that $\phi > 1$ and $\delta < 1$.

(a) Derive the equations of motion describing this firm's behavior and construct an appropriate phase diagram. Label all important features of the diagram.

(b) Prove that this model has a unique stable path, at least near the steady state.

(c) Use your phase diagram to analyze how the firm would respond to an unanticipated temporary fall in 'w'. That is, imagine that the wage suddenly drops and is expected to be low for 4 years, after which it is expected to return to its original level.

(d) Discuss several numerical methods which could be used to obtain quantitative solutions to a model of this type. What are the advantages and disadvantages of each? Which method(s) would be best if this model were to be embedded in a large general equilibrium model?