

Department of Economics  
The University of Texas at Austin

Comprehensive Exam  
**Environmental and Natural Resource Economics**  
January 2002

*Please do all questions.*

**Question 1**

You are to analyze the market for chlorofluorocarbons (CFC), gases like Freon that are used in air conditioners and that deplete the world's stratospheric ozone ( $O_3$ ) layer (which shields the Earth from ultraviolet rays of the sun that cause skin cancer and crop damage). Use  $X$  to denote the number of pounds of CFCs in millions (so that  $X=2$  means two million pounds). The price,  $P$ , is in dollars per pound. You estimate the demand function using income and demographic characteristics, but with respect to price you find that a good approximation is simply  $X=4-P$ . You also estimate that the private cost of production is a constant \$1 per pound (so the total cost is  $\$X$ ). Finally, you estimate that the Total External Cost is  $0.2(X+X^2)$ .

Use this information to answer the following seven questions. For b-g, provide specific numbers. Please list each answer *by-the-letter* (a through g):

- (a) How would you draw the market for this commodity and label the areas?
- (b) What is the unregulated competitive market price and quantity ( $X^C$ ), and what is the socially optimal amount ( $X^*$ )?
- (c) What tax per pound of CFC would lead the private market to  $X^*$ ?
- (d) What is the lost consumer surplus from that tax?
- (e) What is the tax revenue?
- (f) What is the reduction in the external cost of this pollutant?
- (g) What is the net gain in welfare?

**Question 2**

Next we want to discuss some other policy problems and options using more complicated versions of the same example with CFCs. In order not to require the correct answer for #1 just to be able to do #2, however, now assume for simplicity that the Pigovian tax per pound of CFCs would be simply one dollar per pound. Suppose that the pollution does not arise just from the purchase and use of CFCs (fixed pollution per unit), but instead from the *escape* of CFCs into the atmosphere (when the repair person tries to refill the Freon gas, or when the air conditioner is discarded. Again, answer each question by the letter:

- (a) What do you think now about the prospects for using the Pigovian tax solution to this externality? (What is the key feature that makes it different from the tax in #1 above?)

- (b) Suppose instead that a law is passed to “prohibit” the release of CFCs into the air, that violators face a penalty per pound released, and that regulators can effectively find and impose penalty on 20% of the pounds released. What then is the optimal penalty?
- (c) Suppose that this enforcement operation costs a million dollars (not per pound, but a million dollars to do it at all). Now what is the optimal penalty per pound? (This is the one portion of #2 where you might be able to use information derived in #1.)
- (d) What other criteria might further influence your decision about whether and how to recommend this law to policymakers?
- (e) Problem #1 above says that the cost of this pollution (benefit of abatement) is  $0.2(X+X^2)$ . We studied several methods for benefit estimation: which one(s) might work in this case, which one(s) would not work, and why?

### Question 3

A monopolist is extracting an exhaustible resource and faces costs that increase with cumulative extraction. The cost of extracting  $q$  units of the resource when  $x$  units have been extracted to date is equal to  $\gamma q^2 + fx$ , where  $\gamma$  and  $f$  are constants. The demand curve for the resource is given by the following expression and doesn't change over time:

$$p = a - bq$$

where  $a$  and  $b$  are parameters. The stock of  $x$  evolves according to the equation  $\dot{x} = q$ . The firm's objective is to maximize the present value of its profits. If the firm decides to shut down, it will no longer have to pay any costs but it will also not be able to sell the operation to anyone else. The interest rate is constant and equal to  $r$ , and the initial value of  $x$  is zero.

- (a) Set up the firm's maximization problem and take all appropriate first order conditions.
- (b) Show that the firm will shut down at a finite time  $T$ . Please be sure to justify your answer analytically; don't just claim one thing or the other. In addition, explain the result intuitively.
- (c) Solve for  $q(T)$  and  $x(T)$ . Then solve for a closed form expression for  $q(t)$  and use it to sketch the path of  $q$  over time. Finally, obtain an expression that could be solved numerically to obtain the shutdown time,  $T$ , given numerical values of all of the parameters.
- (d) Derive the model's equations of motion in  $(q, x)$  space and construct an appropriate phase diagram. Sketch the trajectory of the model from its initial state.

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*Please do all questions in both parts of the exam.*

**Part I**

The US Congress and the environmental protection agency (EPA) are concerned not only with severe air pollution problems in major cities, but also with “preventing serious deterioration” (PSD) in other more-pristine locations, particularly major national landmarks and tourist attractions. Consider the case of Sedona, Arizona, a small resort town with famous red rock formations, a significant year-round local population, and a growing number of tourist-based time-share condominiums for both winter sports and summer sightseeing. The city does not have smokestack industry, but the canyons trap the growing vehicle air pollution and cause problems viewing the rock formations as well as the usual morality and morbidity damages. You are asked to develop models and data necessary to estimate both costs and benefits of alternative clean-up proposals.

- A. For the measurement of benefits, we have the usual line-up of hedonic house price models, travel cost models, contingent valuation, and direct health cost measures. In this case, however, it seems best to design some way of *combining* two or more of those methods: some live there year-round while others travel from great distances. Some reveal their *use* of this natural resource, and others may have *non-use* benefits from protection. How would you design an *integrated* study of all such benefits?
- B. For the measurement of costs, we have the usual line-up of direct expenditure methods, consumer surplus lost, and computational general equilibrium models that account for multiple price changes and therefore cross-price behavioral effects that might be important if substitute activities and locations are available. Again, you should design some way of *combining* two or more of those methods. How would you design an *integrated* study of all relevant costs?

In both parts A and B, you are already handed the relevant jargon and definitions, so the point here is to go beyond a repeat of the list of methods and think seriously about integration. How can you take the best parts of what each method has to offer, in ways that are consistent with each other and that covers all the necessary costs and benefits.

## Part II

Suppose that a monopolist is extracting an exhaustible resource and faces costs that increase with cumulative extraction. The cost of extracting  $q$  units of the resource when  $x$  units have been extracted to date is equal to  $cqx$ , where  $c$  is a constant. The demand curve for the resource is given by the following expression and doesn't change over time:

$$p = a - bq$$

where  $a$  and  $b$  are parameters. The stock of  $x$  evolves according to the equation  $\dot{x} = q$ . The firm's objective is to maximize the present value of its profits. If the firm decides to shut down, it will no longer have to pay any costs but it will also not be able to sell the operation to anyone else. The interest rate is constant and equal to  $r$ , and the initial value of  $x$  is zero.

- A. Set up the firm's maximization problem and take all appropriate first order conditions.
- B. Determine whether or not the firm will shut down. Discuss. Please be sure to justify your answer analytically; don't just claim one thing or the other.
- C. Derive the model's equations of motion in  $(q, x)$  space. What does this tell you about how  $q$  changes over time? Sketch the trajectory of the model in a phase diagram.

Now suppose there is a backstop technology which is produced by a competitive industry. The backstop is a perfect substitute for the monopolist's product but it is initially very expensive. Over time, the cost falls according to the expression below, where  $a$  and  $\gamma$  are parameters and  $t$  is time. For convenience, let  $a$  have the same value as it had in the demand equation.

$$c_B = a - \gamma t$$

- D. Set up a revised version of the monopolist's problem to take the backstop into account. Think it through carefully and explain why you set up the problem up the way you did. Please note that this will take some thought: it is the hardest part of this problem.
- E. Finally, determine as much as possible about how the monopolist's behavior will change. You probably won't have time to obtain a closed form solution but it is possible to derive some interesting results without going that far. For example, what can you say about the shutdown condition?

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August 2000

*Please do all questions in both parts of the exam.*

**Part I**

Unfortunately for environmental economists (and for the victims of pollution), each type of production usually entails several types of emissions. A single plant may generate some type of toxic waste, other liquid waste, normal industrial garbage for landfill, noise pollution, aesthetic cost, and energy-related emissions of CO<sub>2</sub>, SO<sub>2</sub>, NO<sub>x</sub>, and other air pollutants. How should such a plant be regulated or taxed? Should the damages be combined into a single tax? What if each different type of production produces a different *mix* of such pollutants?

To address some of these questions, write down and solve a model of  $n$  competitive industries that each produce a single output,  $x_i$  ( $i=1, \dots, n$ ) in a constant returns to scale production function. Total resource (land or labor) is  $L$ , which is also numeraire. Each industry may produce any of  $m$  different pollutants. The total output of each pollutant is  $Z_j$  ( $j=1, \dots, m$ ). A single consumer behaves competitively and owns the resource, with utility of the form  $U = U(x_0, x_1, \dots, x_n, Z_1, \dots, Z_m)$ . Good zero is home-use of the resource, which creates no pollution and cannot be taxed. For notational simplicity, use subscripts on  $U$  for partial derivatives, where  $U_i > 0$  for  $i=0, 1, \dots, n$ , and  $U_j < 0$  for  $j=n+1, \dots, n+m$ . Assume that this closed economy has no taxes or other distortions.

(a) Initially, assume constant costs, so a unit of each good can be defined as the amount that

costs \$1 to produce. The resource constraint is then  $L = \sum_{i=0}^n x_i$ . Also assume that each

pollutant bears a fixed relationship to each output. That is, each unit of  $x_i$  generates  $\alpha_{ij}$  units of pollutant  $j$ . Then  $Z_j = \sum_i^n \alpha_{ij} x_i$ . Show that the first best can be achieved with a tax on each output at rate  $t_i$ . Solve for these first-best Pigovian tax rates, and *interpret* them.

(b) Next, suppose that each industry can alter each pollutant per unit of output, in a CRTS production function  $x_i = f^i(L_i, z_{i1}, \dots, z_{im})$ , where  $z_{ij}$  is the amount of pollutant  $j$  generated by industry  $i$ . Then  $Z_j = \sum_i^n z_{ij}$ . Use subscripts on  $f^i$  for partial derivatives, where  $f^i_L > 0$ , and  $f^i_j > 0$  for  $j=1, \dots, m$ . Suppose that each firm faces a private cost of \$1 (one unit of  $L$ ) for each unit of each pollutant, just to cart it away. In this case, the price of each output ( $p_i$ ) may vary as pollution taxes raise the cost of production. Can the first best be achieved using:

1. a tax on each output ( $t_i$ )? Why?
2. a different rate of tax on each pollutant from each industry ( $t_{ij}$ )? Why?
3. just one rate on each pollutant ( $t_j$ ) regardless of source? Why?

Solve for the first-best tax rates in this case, and *interpret* them, as above.

- (c) Can the same first-best allocation be achieved using a system of pollution permits instead of taxes? Why? How many different types of permits are necessary? Can the notation in your tax solution be used to explain the relationship between the tax system and the permit system (that is, to compare the amount of each pollutant, and the cost of each pollutant to each firm)? How would the permits be allocated or sold, to make the two systems equivalent?

## Part II

Please answer **both** questions in this section.

### Question 1

An important aspect of real-world fisheries is that the biological parameters governing the growth rate of the fish stock are not known precisely. This problem asks you to explore how that might affect management of the fishery. Don't be worried by the length of the question – it's a bit long to explain but not very hard to do.

Suppose that a particular stock of fish is known to have a biological growth function of the form  $G(b) = gb(k - b)$ , where  $b$  is the stock of fish and  $g$  and  $k$  are biological constants. The technology of fishing follows the standard Gordon-Shaefer model: when the level of fishing effort is  $e$ , the harvest of fish,  $x$ , is given by the expression:  $x = abe$ , where  $a$  is a technological parameter. Throughout the problem you may assume that  $a=1$ . The price of fish is \$1 and the cost of a unit of effort is  $w$ ; both are exogenous and constant over time.

- (a) Set up an appropriate maximization problem and solve for the optimal sustainable steady state equilibrium. Find explicit expressions for the optimal  $e$ ,  $b$ ,  $x$  and also solve for the annual profit. Be sure to show all your work.

Now let's add the uncertainty. To keep things simple, you can assume that the form of the growth function and value of  $k$  are known and the only thing uncertain is the value of  $g$ . It has been estimated to be  $\bar{g}$  but the estimate has a standard error equal to 10% of its value (that is,  $\sigma = 0.1\bar{g}$ ).

- (b) Suppose that manager has to make a once-and-for-all decision on the number of fish,  $\hat{x}$ , to be harvested each year. The decision cannot be changed later. If the manager wants to be sure that  $\hat{x}$  is not so large that the fish could become extinct, derive an expression for the maximum  $\hat{x}$ . You may interpret "wants to be sure" to mean that the manager wants there to be no more than about a 2.5% chance that the chosen  $\hat{x}$  is too large. Under what circumstances would the manager choose an  $\hat{x}$  that is less than the optimal  $x$  from part (a)? Please be specific.

## Part II

### Question 2

Consider a mine operated by a price-taking firm. Let the price of the mine's output be  $p$ . Although the firm takes  $p$  as given, the demand curve for its output is actually  $p = a - bq$ , where  $a$  and  $b$  are parameters. The firm's cost of extracting  $q$  units is given by:  $cq^2 + cx$ , where  $c$  is an exogenous constant and  $x$  is the cumulative extraction to date.

- (a) Set up the firm's optimization problem and take first order conditions. You may assume that if the firm decides to shut down, it will no longer have any costs but will not be able to sell the mine, either.
- (b) Will the firm shut down, or will it operate forever? Explain how you know. You do not need to derive a closed-form expression for the shutdown time.
- (c) Derive an expression for  $q$  as a function of time, taking into account the demand curve. Sketch the  $p$  and  $q$  trajectories and explain why they look the way they do.

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**Part I**

Your job is policy analyst at the EPA. Your boss is worried about how to reduce air pollution while minimizing economic costs. Currently, a particular industry is not subject to any regulatory restrictions. You describe to her the various conceptual solutions you learned in class: (1) Pigouvian tax or subsidy, (2) quantity regulation, with or without price regulation as well, (3) permits, either handed out or sold by government, and (4) Coase solution, where either party might own the property rights. She asks you to study industry demand and cost functions, diagram them, demonstrate each solution, and explain how it works. You find that the output demanded,  $Q$ , can be expressed as:

$$Q = A + BP + CY$$

where  $Q$  is measured in millions of units (so  $Q=20$  really means 20 million of aggregate demand),  $P$  is the price in dollars per unit, and  $Y$  is aggregate income (in millions of dollars). When this equation is estimated, you find that the coefficients:  $A = 10$  (millions of units),  $B = -1.0$ , and  $C = 0.002$ . Aggregate income is 20,000 (millions of dollars), and it is not expected to change.

Current output is 30 (million units), and the current total cost of production is 600 (millions of dollars). Upon further investigation, you decide that production exhibits no fixed costs, with constant returns to scale, and plenty of competition. You use that information to derive private marginal cost (PMC). Finally, you use estimates (from studies of health effects, visibility, etc.) to find that marginal external cost (MEC) is  $Q/2$ . In other words, the first unit of output (at  $Q=0$ ) has no environmental cost at all, but at the current output level ( $Q=30$ ), one more unit of output imposes \$15 of external cost (assuming at each level of output that nobody can change the external cost per unit of output). Use all of this information together to draw PMC, MEC, SMC, Demand, current (competitive) output  $Q^c$ , and the socially optimum output  $Q^*$ .

Analyze each of the four solutions to the externality problem, each with version A and B. For each solution, describe: What assumptions are required for this solution to work perfectly? With those assumptions, how does that policy redirect the economy from the competitive outcome to the optimal outcome? What is the dollar value for five outcomes: the change in consumer surplus, the change in government revenue, the change in industry profits, the change in environmental damages, and the net change in welfare. You can provide these five results for each of eight solutions (1A to 4B) in the attached matrix with 40 entries (*see the last page of the*

*exam*). If you use this matrix, please put your exam number on it and attach it to your written answers. Which policies are equivalent in this simple model?

Next, relax some of the assumptions of that model to discuss how the policies differ in other ways. Describe problems with each type of solution, using examples(s) from actual environmental policies in the United States (or elsewhere) that deal with air pollution, water pollution, hazardous waste, or toxic substances. How is each type of policy enforced? What information is required? What is the meaning of “cost-effectiveness analysis”? Apply this concept to the choice between a command-and-control policy (CAC, like the quantity restriction) and an incentive-based policy (IBP, like taxes or permits)? Which is likely to be more cost-effective, and why?

## Part II

Suppose that a previously-untouched rainforest is being converted into agricultural land by a large group of competitive firms. To keep things simple, we’ll treat them as a single aggregate firm that takes prices as given. The firm receives a one-time payment of  $p$  dollars for each hectare it converts. The cost of converting (bulldozing)  $b$  hectares is  $cb^2$ , where  $c$  is a constant. Thus, at each instant in time the firm’s profit is:

$$\pi = pb - cb^2$$

The rainforest, however, provides an externality in the form of biodiversity. The level of biodiversity is measured by the number of species,  $s$ , living in the forest and  $s$  is reduced when the land is converted. What’s more, biologists who have studied the area have found that the faster the land is converted, the greater the loss of biodiversity:

$$\dot{s} = -b^2$$

The social value of biodiversity is  $hs$ , where  $h$  is a constant. The initial level of biodiversity is  $s_0$ , and you may assume the interest rate is constant at  $r$ .

- (a) Set up the firm’s optimization problem when it ignores biodiversity and solve for its behavior. Find an expression for  $b$  as a function of time and solve for the time  $T$  at which biodiversity has been driven to zero.
- (b) Now set up the social planner’s problem (taking the externality into account) and find the first order conditions.
- (c) Solve for the equations of motion in  $(\lambda, s)$  space, where  $\lambda$  is the costate variable, and draw an appropriate phase diagram. Be sure to label everything carefully. Discuss anything unusual about it.

- (d) Now find an equation for the optimal path of  $b$  as a function of time and then draw an appropriate sketch. Explain it in detail and discuss any noteworthy features (including how it compares to the competitive outcome.)
- (e) Finally, draw another phase diagram and use it to analyze the effects of a temporary increase in  $h$ . Sketch the time paths of  $s$ ,  $\lambda$  and  $b$  and explain what's going on in words.

Please attach this to your answer.

Exam number: \_\_\_\_\_

Numerical answers can be provided in the following matrix, which must be attached into your submitted answer. These are not complete answers, however, as the questions also ask for a diagram, descriptions of how each solution works, and examples of some of the problems with each.

Conceptual Solution (provide "name")	Change in Consumer Surplus	Change in Government Revenue	Change in Industry Profits	Change in Environmental Damage	Net Change in Welfare
1A.					
1B.					
2A.					
2B.					
3A.					
3B.					
4A.					
4B.					

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*Please do all parts of both of the following questions.*

**Question 1**

Using (and listing) all of the perfect assumptions, set up a simple static model for production of an agricultural output that requires using up some labor and using up some forest. In other words, per-capita output is  $X=F(L_X,T)$ , where the two inputs are  $L$ =labor and  $T$ =trees cut. Each unit of deforestation requires one unit of labor ( $T=L_T$ ). Then  $N$  homogeneous consumers each get utility  $U(X,H,D)$ , where  $D=NT$  is total deforestation and  $H$  is home production. Each unit of  $H$  is produced using one unit of labor,  $H=L_H$ . Total labor is fixed ( $L=L_X+L_H+L_T$ ). While  $X$  and  $H$  each have a positive effect on utility,  $D$  has a negative effect.

- (a) Showing all steps, derive a set of first-best tax rates on each good ( $X$ ,  $H$ ,  $T$ ,  $L_X$ ,  $L_Y$ ,  $L_T$ ).
- (b) Now suppose that government cannot tax  $H$ , but needs a fixed amount of revenue to provide a public good ( $G$ , which enters utility in a separable fashion and which is produced using  $NL_G$ ). How would you set up a model to derive the second-best tax rates? What would that model be likely to show? How would you interpret those results?
- (c) To implement the tax rate in either part (a) or (b), the formula requires knowledge of the marginal effect of deforestation on utility. What are some possible components of that effect? That is, what aspects do people value about forests? What are a few of the options for methods to measure each such value? What are some of the pitfalls of each such method? For each such value, which method would you choose to measure it, and why?

## Question 2

Suppose that a price-taking firm is extracting oil from an offshore platform and is subject to increasing costs as the stock of oil is depleted. At time  $t$  the firm's profit is given by:

$$\pi = pq - cq^2 - ex$$

where  $p$ ,  $c$  and  $e$  are exogenous constants,  $q$  is the firm's extraction rate, and  $x$  is the amount of the resource extracted to date.  $\pi$ ,  $q$  and  $x$  are all functions of time and  $x$  evolves according to the following equation:

$$\dot{x} = q$$

Government regulations require the firm to remove the platform when oil extraction ceases. Removing the platform requires a lump sum payment of  $D$  at the time the platform is removed.  $D$  is exogenous and does not depend on  $x$ . The firm wants to maximize the present value of its profits taking into account that it will eventually have to pay  $D$ . Let the interest rate be  $r$  (you may assume that it is constant).

- (a) Set up the firm's optimization problem and take first order conditions. Be sure not to forget about the terminal conditions.
- (b) Solve for the values of  $q(T)$  and  $x(T)$  at the time  $T$  that the firm shuts down the platform. How does  $x(T)$  compare to what the firm would choose if it didn't have to pay  $D$ ? Explain intuitively why this happens. Please discuss this as thoroughly as you can.
- (c) Solve for  $q(t)$  in terms of the exogenous variables and the ending time  $T$ . Draw a graph of it.
- (d) Derive an implicit equation for  $T$  in terms of the exogenous variables. You do not need to solve for  $T$  explicitly.
- (e) Derive the equations of motion for the problem in  $(q,x)$  space and draw an appropriate phase diagram showing the trajectory of the firm from the initial point to the terminal point. Be sure to label everything in the diagram.

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*Please do any THREE of the following four questions.*

**Question 1**

Consider a closed economy in a static model with many identical farmers that produce  $X$  using capital  $K_X$  and water  $W$  in a constant returns to scale production function  $X = F(K_X, W)$ . Each can pump water from the single aquifer and obtain water directly, with no fixed cost and constant marginal cost in terms of labor. Define a unit of water  $W$  as that which can be obtained using one unit of time, so that  $W=L_W$ . Another output  $Y$  can be produced using capital and labor in  $Y=G(K_Y, L_Y)$ .

Also,  $N$  identical consumers have utility  $U=U(X, Y, H, T)$ , with positive marginal utility from farm output  $X$ , other output  $Y$ , home production  $H$ , and the level of the water table  $T$ . Define a unit of  $H$  as that which uses one unit of time, so  $H=L_H$ . With fixed capital  $K$  and labor  $L$ , the economy is constrained by  $K = K_X + K_Y$  and  $L = L_X + L_W + L_Y + L_H$ . Finally, the water table depends on total use of water,  $T= T(NW)$ , with negative derivative  $T'$ .

Does the resulting market equilibrium attain a social optimum? Why or why not? If not, how could the problem be corrected? Suppose the government can use lump sum taxes and rebates. If desired, taxes can be applied to output of  $X$  at rate  $t_X$ , output of  $Y$  at rate  $t_Y$ , use of capital in  $X$  at rate  $t_{K_X}$ , and/or use of capital in  $Y$  at rate  $t_{K_Y}$ . In contrast, taxes on labor have two problems. First, home production cannot be observed, so any tax on market labor would distort labor supply. Second, the resources devoted to pumping on ones own property cannot be observed. The authorities cannot accurately monitor the water taken by each farmer, so they cannot impose a tax per unit of  $W$ . Can the available tax rates be used within competitive markets to achieve the first-best social optimum? If the answer is yes, how would you set each of these tax rates? If the answer is no, and labor must be taxed, then how would you characterize the second-best social optimum?

## Question 2

Design a relatively-low-cost study to measure the cost of environmental damages from the accidental contamination of a particular stretch of a particular river called “The Rapids”. This stretch has long been used for river rafting, but it must be closed for two years because a factory upstream has spilled a chemical. You are hired by the state to provide an expert estimate of damages, for use in a suit against the owner of the factory, but the state provides only a limited budget that does *not* allow you to mount your own new data collection such as any new survey of residents. How would you proceed? Would it be sufficient to use existing data on The Rapids (such as entry fees paid in each prior year)? Why? Would you be able to use other existing studies of other similar locations? What kinds of other existing studies would you need, how many such studies would you need, and how would you make use of them?

## Question 3

Suppose that in the absence of fishing, the natural growth of the population of tuna,  $b$ , is given by the equation:

$$\dot{b} = b - 0.001b^2$$

In addition, suppose that  $e$  units of fishing effort produce a tuna harvest,  $x$ , given by  $x=be$ . The cost of each unit of effort is \$100. The price of tuna,  $p$ , depends on the amount harvested and is given by  $p=80/x$ . There is free entry into the industry and firms take prices as given. The rate of change of effort is proportional to current profits,  $\pi$ :

$$\dot{e} = 0.1\pi$$

- Derive the equations of motion for the system in  $(b,e)$  space and solve for the steady state values of  $b$ ,  $e$ ,  $x$  and  $p$ .
- Solve for the model's eigenvalues near the steady state and discuss the system's stability thoroughly.
- Draw an appropriate phase diagram, placing  $b$  on the horizontal axis. Label everything and show how the system might move to the steady state from an initial equilibrium with no fishing. Show the trajectory as accurately as you can.
- Now solve for the efficient steady state outcome assuming that entry could be restricted by the government. Explain your reasoning and calculate the values of  $b$ ,  $e$ ,  $x$  and  $p$ .

#### Question 4

Suppose the profits of operating a particular gold mine at time  $t$  are given by the expression:

$$\pi_t = pq_t - \frac{cq_t^2}{x_t} - f$$

where  $p$  is the price of gold (which the mining firm takes as given and is constant over time),  $q$  is the amount of gold extracted at time  $t$ ,  $c$  is a constant cost,  $x$  is the amount of gold remaining in the mine, and  $f$  is a licensing cost that must be paid to the government whenever the mine is in operation. The firm's goal is to maximize the present value of profits on the mine. The evolution of  $x$  is given by:

$$\dot{x}_t = -q_t$$

The initial amount of gold,  $x_0$ , is known to be very large. Finally, you may assume the interest rate is zero.

- (a) Determine whether the mine will operate forever or will eventually shut down. Be sure to prove your answer, don't just assert one or the other.
- (b) Solve for the model's equations of motion in  $(q, x)$  space and sketch a reasonable trajectory for the optimal path.
- (c) Set up the model as a discrete-time dynamic programming problem and solve backwards from your result in (a) for two periods.
- (d) Explain in words how you would proceed from (c) to a complete solution of the model.

Department of Economics  
The University of Texas at Austin

Comprehensive Exam  
**Environmental and Natural Resource Economics**  
August 1998

*(Please answer both parts A and B.)*

**Part A**

An interesting and important new application of natural resource economics is the study of antibiotic resistance. To see how this works, consider a population  $P$  that is affected by a bacterial disease. To make things simple, assume that each person in the population is either (1) well, (2) sick with the normal form of the bacteria, which is susceptible to antibiotics, or (3) sick with an antibiotic-resistant bacteria. Let the number of people who are well at time  $t$  be  $W$ , the number of people who are sick with the normal bacteria be  $N$ , and the number sick with the resistant bacteria be  $R$ . You may assume the disease is not fatal ( $P$  is constant) and that  $W = P - N - R$  at all times.

Suppose that at each point in time, the number of new cases of the normal disease will depend on  $W$  and  $N$  as follows:

$$\phi_N (WN)^{1/2}$$

where  $\phi_N$  is a parameter. The number of new cases of the resistant form is similar:

$$\phi_R (WR)^{1/2}$$

The two forms of the disease are identical except that the resistant form of the disease is less contagious:  $\phi_R < \phi_N$ . When left untreated, people recover from both forms of the disease at rate  $\delta$ . In other words, at each point in time natural recovery reduces the stock of people with the normal disease by  $\delta N$  and the stock with the resistant form by  $\delta R$ . You may assume that  $\delta$  is much larger than  $\phi_N$  or  $\phi_R$ .

1. Write down the equations of motion for this system in terms of  $N$  and  $R$  and the exogenous variables and parameters. Solve for the steady state fractions of the population who are in each state (e.g.,  $W/P$ , etc.) in terms of the parameters.
2. Construct an appropriate phase diagram for the system. Put  $N$  on the horizontal axis and show the steady state, the isoclines and the directions of motion. Please show your work and label everything carefully and clearly. *In addition*, please draw a line showing points in the plane with the same total number of cases of the disease as the steady state. Be sure to get its slope right relative to the isoclines.

3. Discuss the stability of the model near the steady state.

Now let's add antibiotics to the model. Suppose that antibiotics cause a person to get well immediately with probability  $\epsilon$  if she has the normal form of the disease and has no effect if she has the resistant form. However, the decision to administer the antibiotic must be made before it is known which form of the disease she has.

4. Write down the new equations of motion when all sick people are given antibiotics. Solve for the new steady state and discuss how it compares with the old one. Was it a good idea to introduce antibiotics? Why?
5. Draw a new phase diagram showing the old and new steady states and a trajectory the model might follow from the old steady state when antibiotics are introduced.
6. Discuss how the path from part (5) would be viewed by the public. How would antibiotics look in the short run? How would they look in the long run? What would you expect to hear from doctors and hospitals during the transition between the short and long run?

*That's the end of the question.* For whatever it's worth, an important aspect of the real world that has been left out of this model for tractability is that some people who are given antibiotics stop taking them before they are completely well. That tends to accelerate the development of resistant bacteria. One way to think about this would be that giving someone antibiotics to someone with the normal disease causes them to get well with probability  $\epsilon$  and causes them to develop the resistant strain with probability  $\gamma$ . This would be an interesting research topic but you don't need to say anything about it on this exam.

## Part B

The European Union is concerned about Sulfur Dioxide (SO<sub>2</sub>) emissions of electric power plants, vehicles and industrial sources. They are debating two major types of choices. First, they can choose among several methods of control, including an Emission Permit System (EPS), Ambient Permit System (APS), Pigouvian Tax, or command and control (CAC) in the form of a performance standard or technology standard. Second, they can choose whether the problem should be left to local levels of government, national levels, the EU as a whole, or a global system. For *each* of the following facts, separately, describe how that fact would affect the best decision for *each* of these two kinds of choices (the choice of policy, and the choice of level):

- (a) The jet stream blows from west to east.
- (b) Vehicle emissions never reach high enough to enter the jet stream.
- (c) To comply with local ambient requirements, smokestacks are getting taller.
- (d) SO<sub>2</sub> emissions are generally associated with simultaneous emissions of other pollutants such as CO, NO<sub>x</sub>, VOC and CO<sub>2</sub>.
- (e) Vehicle emissions cannot effectively be monitored for each vehicle.
- (f) The marginal benefits of abatement (MBA) cannot be measured with certainty.
- (g) The MBA is steeply sloped, rather than flat.
- (h) The marginal cost of abatement (MCA) cannot be measured with certainty.
- (i) The MCA is steeply sloped, rather than flat.
- (j) Pricing in the electric power industry is regulated locally by the European equivalent of the U.S. Public Utility Commissions (PUC).
- (k) Workers in the electric power plants have industry-specific training in skills that are not useful elsewhere.
- (l) Electric power is traded within the EU but cannot effectively be transported further for international trade.
- (m) Cultural differences as well as transportation costs prevent labor from inter-regional migration.
- (n) All of Europe already suffers from high levels of taxation on labor income.

Comprehensive Exam  
**Environmental and Natural Resource Economics**  
August 1997

*Please answer all four questions.*

**Question 1**

Suppose that in the absence of human intervention the population of swordfish grows at the rate:

$$\dot{b} = g(k - b)$$

where  $g$  and  $k$  are constants and  $b$  is the biomass of swordfish. Further, suppose that the price of one unit of harvested swordfish is  $p$  and that the cost of harvesting  $x$  units is:

$$c(x, b) = wx\left(1 + \frac{x}{b}\right)$$

where  $w$  is the wage rate.

- (a) Suppose the industry is controlled by a single firm whose objective is to maximize the present value of the profits it earns. Set up the firm's optimization problem and derive its first order conditions. You may assume the firm takes  $p$  and  $w$  as given.
- (b) Solve for the steady state values of  $x$ ,  $b$  and the costate variable when  $g=0.05$ ,  $k=10$ ,  $p=11$ ,  $w=10$ , and  $r=0.05$ .
- (c) Construct a phase diagram representing the model's equations of motion. Put the state variable on the horizontal axis and the costate on the vertical axis. Show the steady state, the isoclines and the directions of motion in each quadrant.
- (d) Determine the stability properties of the model. Use this result to find the locus of possible starting points in the phase diagram from which the model will converge to the steady state.
- (e) Now construct an alternate phase diagram in which  $x$  is on the vertical axis and  $b$  is on the horizontal axis. Show the locations of any stable paths.

## Question 2

Suppose an economy possesses a single resource: an area of undeveloped land. The land may be left in its natural state or it may be converted and used for manufacturing. Let the stock of undeveloped land at time  $t$  be  $A(t)$  and let the initial endowment of undeveloped land be  $A_0$ . The evolution of  $A$  is governed by the equation:

$$\frac{dA}{dt} = -x$$

where  $x(t)$  is the rate at which land is being developed at time  $t$ . The quantity of manufactured goods that can be made at any time depends on how much land has been converted to manufacturing,  $A_0 - A(t)$ , and is augmented by technical change at rate  $\mu$ . Let  $M(t)$  indicate the amount of manufactured goods at time  $t$ , and let the production function for  $M$  be given by:

$$M(t) = (A_0 - A(t))e^{\mu t}$$

Finally, assume that consumers like both manufactured goods and natural land, and that their preferences can be represented by an agent with the following utility function:

$$U = \int_0^{\infty} \ln(\min\{M(t), A(t)\})e^{-\rho t} dt$$

- (a) Assuming that land can be converted between uses at no cost, and that conversions are not irreversible, find the optimal trajectories for  $A$  and  $x$ .
- (b) Now suppose that conversions are irreversible: once land has been developed it can never be turned back into a park. Explain in words (and in detail) how this affects the problem.
- (c) Derive an equation that could be solved numerically to find the optimal trajectory for  $A$  when conversions are irreversible. (This will be a transcendental equation so you won't be able to solve for a closed-form equation for  $A$ .) Hint: you'll need to think carefully about what you learned from (a) and about how the  $\min\{M, A\}$  function works.

### Question 3

Set up a simple general equilibrium model that can be used to solve for the Pigovian tax on emissions. To do this, first list the “perfect” assumptions, and assume  $n$  identical individuals. Suppose utility per capita is  $u=u(x,q,Z)$ . Suppose production of each unit of  $x$  uses one unit of resources  $r$  (such as labor). Production of  $q$  uses resources (amount  $y$ ) and emissions (amount  $z$ ) in a production function  $q = f(y,z)$ . With constant returns to scale, these lower case letters can all represent per capita amounts. One unit of emissions itself requires one unit of resources (for transport and disposal). Aggregate emissions  $Z=nz$  enter negatively into utility. Last, the household’s resource  $r$  is numeraire and is constrained by  $r = x + y + z$ .

- (a) Assume that lump sum taxes are available, so we can ignore the government’s revenue requirement. For simplicity, leave out all tax rates but  $t_z$ . Show all the steps to solve for the first-best Pigovian tax rate.
- (b) Do not solve explicitly, but describe how the answer above would be modified by consideration of other distorting taxes (such as on individual supply of resources like labor), in a model with no lump sum taxes and a government revenue requirement.

#### Question 4

The regional government is contemplating a major new dam and water project that would create substantial costs and benefits. Some of these costs and benefits appear in the first list below. The second list includes methodologies to measure the value of a particular cost or benefit. How would you match up the two lists? For each cost or benefit on this first list, which methodology on the second list is best suited to measure it, and why? For each item on the first list, BRIEFLY outline how the chosen method would be applied.

- I. The dam will provide a fish ladder, but will still interfere with the travel of adult salmon upstream to spawn, and of baby salmon downstream. Some of these salmon species are “threatened.”
- II. The dam will provide hydroelectric power, will reduce the burning of fossil fuel, and will thus reduce damaging emissions of carbon monoxide, particulates, and other air pollutants in the area.
- III. The dam will require inputs of labor, concrete, steel, bulldozers, and other materials.
- IV. The project will result in a lake for fishing, swimming, and boating.
- V. The project will provide greater insurance against flooding.

Possible methodologies include the use of:

- A. Market prices
- B. Hedonic regressions
- C. Contingent Valuation
- D. Medical cost of health effects
- E. Travel Cost method.

Department of Economics  
The University of Texas at Austin

Comprehensive Exam  
**Environmental and Natural Resource Economics**  
August 1996

*Please answer all of the following questions.*

**Question 1**

Consider a model of the second-best like that of Bovenberg and de Mooij (1994 AER), a simple analytical general equilibrium model with one consumer, one period, one factor of production, a clean good, a dirty good, a labor-leisure choice, a pre-existing labor tax, and an uncorrected externality that is fixed per unit of the dirty good. First, restate the basic results of B&dM about the second-best optimal tax on the dirty good, and explain the intuition. Second, apply the model to the case where the externality is addressed not by a tax on the dirty good, but by a command and control (CAC) type policy that might restrict the quantity of the dirty good. (You may use partial equilibrium diagrams to explain your *intuition* for the likely result of such a policy, but remember that the question necessarily involves a general equilibrium answer.)

**Question 2**

What determines whether you should model an environmental problem as an externality, or as a public good? What is the difference? Cite examples in the literature. Also, pick an actual policy problem, and explain how and why you would model it (either as an externality, or as a public good). Then explain what could be different about the problem that might make you model it the other way (and how the results would be expected to differ).

### Question 3

Whales are an unusual resource because they are valuable dead or alive. On one hand, they can be caught and used commercially, which produces profits. On the other hand, people seem to attach a high value to the existence of whales in the wild. Suppose the profits on a harvest of  $x$  whales is given by  $\pi = px - c(x, b)$ , where  $p$  is the price of a whale (exogenous in this problem) and  $c(x, b)$  is a function giving the cost of catching  $x$  whales when the stock of whales is  $b$ . In addition, suppose the existence value of  $b$  whales is equal to  $hb$ , where  $h$  is a parameter. Finally, let the evolution of the whale stock be given by  $db/dt = G(b) - x$ , where  $G(b)$  is a function giving the net reproduction of whales when the stock is  $b$ . The interest rate is  $r$ , and it is constant.

- (a) Show that the optimal use of whales must satisfy the following equations:

$$p - \frac{\partial c}{\partial x} = \lambda$$

$$\frac{d\lambda}{dt} = (r - \frac{dG}{db})\lambda + \frac{\partial c}{\partial b} - h$$

$$\frac{db}{dt} = G(b) - x$$

- (b) Now suppose that  $c(x, b) = wx^2 - ab$ , where  $w$  and  $a$  are exogenous variables. Show that the model can be represented by the following equations of motion in  $(x, b)$  space:

$$\frac{dx}{dt} = \frac{1}{2w} \left( a + h - (r - \frac{dG}{db})(p - 2wx) \right)$$

$$\frac{db}{dt} = G(b) - x$$

- (c) Now show that under reasonable assumptions about  $G(b)$  (be specific about what these are), if people suddenly become concerned about the existence of whales ( $h$  goes from zero to a positive number), the optimal steady state value of  $b$  will increase.
- (d) Construct an appropriate phase diagram for the model *with  $x$  on the vertical axis*. Show the steady state, the isoclines, the directions of motion and the stable path. Label everything.
- (e) Suppose it becomes temporarily fashionable to be concerned about whales. Construct a phase diagram showing how the stock of whales and the harvest should evolve for a temporary increase in  $h$ . You may assume that the model is initially at the steady state, and that the duration of the fad is known in advance. Draw appropriate integral curves and discuss your results.

Department of Economics  
The University of Texas at Austin

Comprehensive Exam

**Environmental and Natural Resource Economics**

January 1996

*This exam has three questions. Please answer all three.  
Note that Question 1 counts twice as much as the other two.*

**Question 1 (50 points)**

Global warming is caused in part by the accumulation of carbon dioxide (CO<sub>2</sub>) in the atmosphere. Unlike many other air pollutants, CO<sub>2</sub> is a stock pollutant: once emitted it remains in the atmosphere for hundreds of years. Moreover, global temperature is an increasing function of the stock of CO<sub>2</sub>. This problem asks you to consider how such a pollutant should be managed.

Suppose that the benefits of emitting  $x$  tons of CO<sub>2</sub> are equal to  $bx(t) - hx(t)^2$ , where  $x(t)$  is emissions at time  $t$ , and  $b$  and  $h$  are constants. Also suppose that the damage caused by higher temperatures can be expressed as a linear function of the stock:  $cs(t)$ , where  $s(t)$  is the stock and  $c$  is a constant. Thus, the net present value of the CO<sub>2</sub> emissions path is given by:

$$\int_0^{\infty} (bx - hx^2 - cs)e^{-rt} dt$$

where  $r$  is the interest rate. The evolution of  $s$  is governed by  $ds/dt = x - \delta s$ , where  $\delta$  is the rate at which excess CO<sub>2</sub> is naturally removed from the atmosphere.

- (a) Show that the optimal trajectory of the model in  $(s,x)$  space is described by the following equations of motion:

$$\begin{aligned} \frac{dx}{dt} &= (r + \delta)x - \frac{1}{2h}((r + \delta)b - c) \\ \frac{ds}{dt} &= x - \delta s \end{aligned}$$

- (b) Solve for the steady state and construct an appropriate phase diagram *with  $x$  on the vertical axis*. Show the steady state, the isoclines, the directions of motion and the stable path. Label everything.

- (c) Prove that this system is saddle-path stable.
- (d) Compare the efficient trajectory with the path the model would follow if private agents choose emissions levels ignoring the stock effect. (*Be specific, don't just say emissions would be "higher" or "lower".*)
- (e) Suppose it is suddenly discovered that the damage from global warming is much larger than previously thought. Draw an appropriate phase diagram and show how the path of CO2 emissions should change.
- (f) What would be the optimal path of  $x$  and  $s$  if CO2 were never cleared from the atmosphere (that is,  $\delta = 0$ ). Discuss.

### **Question 2 (25 points)**

Describe several methods that might be used to measure the costs or benefits of non-marketed environmental amenities. Explain how you would measure each of the following: (1) the value of a "wild and scenic" river (such rivers are currently protected by federal law), (2) the costs of global warming, (3) the externality costs of a new pesticide, (4) the costs of "brownfields" (abandoned industrial sites in urban areas that might be contaminated with hazardous wastes).

### **Question 3 (25 points)**

Environmental problems can be addressed by many levels of government -- local, regional, national, international -- or there can be no intervention at all. Discuss how one might decide which level is appropriate for a particular problem. Refer to the literature wherever possible.

Department of Economics  
The University of Texas at Austin

Comprehensive Exam

**Environmental and Natural Resource Economics**

September 1995

*Please answer each of the following four questions.  
Some of the questions have several parts.*

**1. Part 1 (60 points)**

- (a) The debates over NAFTA and GATT included a great deal of discussion about harmonizing environmental regulations (making them more similar) between different countries. Is this a good idea? Discuss. What does the US experience suggest?
- (b) Is there any reason why one might prefer to control an externality with a quantity-based method such as tradable permits instead of a price-based method such as a tax? Discuss?
- (c) Environmental policy becomes more difficult in a second-best world, or in the presence of nonconvexities. Discuss how these factors affect the problem.

## 2. Estimating Benefits (30 points)

The Austin airport is going to be moved from its current location to the Bergstrom Air Force base, in part to reduce noise in East Austin. Explain in detail how you might use hedonic pricing to calculate the value of the move. Assume that noise is the only externality associated with the airport, and that no one will be bothered by noise when the airport has been moved.

In answering this question, you may assume you have access to the kinds of data the government would ordinarily have collected, such as data on the location, price, and selling date of all houses sold in the city during the last five years, and real estate listings giving the physical features of all houses placed on the market during the same period of time (number of bedrooms, fireplaces, square footage, school district, address, etc.). In addition, assume you have a city map with contour lines showing both the average daily noise caused by aircraft with the airport at its current and new locations; and a similar map showing the average household income in different neighborhoods. You do not, however, have demographic or income data on the individual buyers or sellers of houses.

The phrase "explain in detail" means that you should discuss at least the following things. What regression equation(s) you would estimate? Explain your choice of variables and functional forms. How does the length of your data set affect things? Does the unobserved heterogeneity of the households matter? How would you deal with that? How would you use your regression results to compute the overall value? Would it be worthwhile to use any other valuation methods? Why or why not?

### 3. Aquifers and Amenities (45 points)

Suppose you are given the task of managing an aquifer like the Edwards, which produces amenity benefits in addition to water. You have the following information:

Demand for water (excluding amenity value):  $P(t) = A - BQ(t)$

Value of the amenity benefits:  $FX(t)$

Cost of extracting  $Q$  acre feet:  $C(S - X(t))Q(t)$

Evolution of the aquifer:  $dX/dt = R - KQ(t)$

Capacity of the aquifer:  $X(t) \leq S$

where  $A$ ,  $B$ ,  $C$ ,  $S$ ,  $F$ ,  $R$  and  $K$  are constants with the usual interpretations,  $Q(t)$  is the rate of extraction and  $X(t)$  is the height of water in the aquifer.

- (a) Solve for the optimal steady state  $Q$  and  $X$ . Draw and label an appropriate phase diagram.
- (b) Suppose amenities become temporarily more valuable. Using an appropriate phase diagram discuss how that would affect the optimal pattern of water use.

## 1. Mining (45 points)

A perfectly competitive firm extracts ore from a mine. The firm maximizes the undiscounted value of its profit stream, which is given by the following (the interest rate is zero to keep the arithmetic simple):

$$\int pq(t) - cq(t)^2 - fx(t)$$

where  $p$  is the price of ore (exogenous and constant),  $q(t)$  is the quantity of ore extracted at each instant of time,  $c$  and  $f$  are constants, and  $x(t)$  is the total amount extracted from the mine to date. The firm is subject to the following constraints:

$$\dot{x}(t) = q(t)$$

$$x(0) = 0, \quad x(t) \leq A$$

$$q(t) \geq 0$$

where  $A$  is the initial stock of ore.

Determine what the firm does as completely as you can. Explain your results and draw an appropriate phase diagram.

Department of Economics  
The University of Texas at Austin

Comprehensive Exam

**Environmental and Natural Resource Economics**

January 1994

*Please answer each of the following four questions.  
Some of the questions have several parts.*

**1. Instruments for Environmental Policy**

- (a) Compare and contrast command and control regulation with market based policies for dealing with environmental problems.
- (b) Are all market based mechanisms equally satisfactory in all circumstances? Discuss two market based policies and explain as thoroughly as possible when one policy might be preferred to other.
- (c) US ambient air quality regulations are set at the national level. Is this likely to be efficient? Discuss.
- (d) Discuss the US experience with marketable permit systems.

**2. Estimating Benefits**

- (a) Describe and critique the principal methods available for estimating the value of nonmarketed goods such as environmental amenities.
- (b) An economist wants to use the travel cost method to estimate the marginal benefit of increasing water quality in reservoirs used for recreation. He observes two reservoirs, one with water quality (denoted by variable  $Z$ ) equal to 10 and the other with water quality equal to 20 (that is,  $Z=10$  and  $Z=20$ , where larger numbers are better). In the absence of entry fees, the number of visitors to the reservoirs are 100 and 200, respectively. Using the travel cost method, the economist calculates that if an entry fee of \$10 were imposed at each reservoir, the number of visitors would fall to 90 and 190. Neither reservoir is congested.

Assuming that the demand curves are linear (that is, they have the form  $P = a - bQ$  where  $a$

and  $b$  are constants that may differ across the reservoirs), that the reservoirs are identical except for water quality, that they do not compete for visitors, and that income effects are negligible, construct an expression for the marginal benefits of water quality as a function of  $Z$ .

### 3. Fishery Management

Suppose that in the absence of human intervention the stock of salmon ( $b$ ) would change according to the following equation:  $db/dt = 2000b - b^2$ . Furthermore, suppose  $y$  units of fishing effort will produce a harvest of salmon ( $x$ ) given by  $x = by$ . Finally, suppose the cost of  $y$  units of effort is  $\$200y$ , where  $\$200$  is the wage rate, and that the price of salmon is constant at  $\$1$  dollar per unit of  $x$ .

- (a) Find the profit-maximizing sustainable equilibrium for this model when the salmon are *not* common property. Solve for the levels of  $b$ ,  $y$ ,  $x$  and profit.

Now suppose the government allows free entry into the salmon fishing industry but imposes two ad valorem taxes:  $t^x$  on the harvest, and  $t^p$  on profit. Profit now becomes:

$$\pi = [ \$1(1 - t^x)x - \$200y ](1 - t^p)$$

Finally, assume people in the industry behave in such a way that the level of fishing effort can be described by the following differential equation:  $dy/dt = 2\pi$ .

- (b) Calculate the long run equilibrium levels of  $b$ ,  $y$ ,  $x$  and  $\pi$  when both tax rates are set to zero. Evaluate this outcome relative to the one from part (a).
- (c) Suppose the government wants to use  $t^x$  and  $t^p$  to improve the outcome in the industry. Find a tax policy (that is, settings for the two taxes) that would move the long run market outcome to the efficient steady state from part (a). Discuss your results.
- (d) Now find a tax policy that would accomplish the goal in part (c) *and* would insure that for points near the steady state the industry would move toward the steady state without oscillations. Explain your results.
- (e) Draw a phase diagram for the model from part (d). Put the level of effort on the horizontal axis. Show the steady state, the isoclines and the directions of motion. Also, show paths from starting points in two different quadrants to the steady state. Sketch the integral curves for these paths.

### 4. Mining

Suppose that the profit on an aluminum mine at time  $t$  is given by  $\pi = pq_t - cq_t^2 - ks_t$ , where  $p$ ,  $c$ , and  $k$  are exogenous constants,  $q_t$  is the amount of aluminum extracted in period  $t$ , and  $s_t$  is the total amount of aluminum extracted in all periods before  $t$ . The evolution of  $s$  is given by  $ds/dt =$

$q$ . The value of  $s$  is initially zero. Also, the resource is infinite supply in the sense that there is no upper limit on  $s$ . The firm's goal is to maximize the present value of profits it earns on the mine. For convenience, you may assume that the interest rate is zero. Set up and solve the firm's optimization problem. Graph  $q$  and  $s$  as functions of time. Discuss your results.