

Exercise 4

Consider an economy in which the intertemporal behavior of households can be captured by a utility function of the following form:

$$U = \int_0^{\infty} \ln(u) e^{-0.05s} ds$$

where u is the instantaneous utility, or felicity, received by the households at each point in time. It is a function of the households' consumption of goods x and y , C_x and C_y , as shown below:

$$u = C_x^{0.5} C_y^{0.5}$$

Households have an endowment of labor, L , which they supply inelastically. Output of x and y is linear in labor and capital, respectively: $Q_x = L$ and $Q_y = K$. In addition to being consumed, good x (the good produced using labor) is used for investment. The government collects and consumes T units of good y as a tax. The capital stock evolves according to the following accumulation equation:

$$\frac{dK}{dt} = I - 0.1K$$

where I is the quantity of good x used for investment.

Please answer the following questions.

- (1) Derive the equations of motion for the model and find the steady state.
- (2) Construct a phase diagram for this model in (λ, K) space, where λ is the costate variable. Be sure to show the steady state, the isoclines, the directions of motion and the stable path. Show all your work and label everything appropriately.
- (3) Now make a copy of the diagram and use it to analyze a temporary increase in T taking place immediately and lasting for four years. In addition, draw appropriate integral curves for λ , K , I , C_x and C_y . Explain why the C_x and C_y trajectories look the way they do.