

Exercise 5

Consider an economy composed of two households, two firms and a government. Let the households be denoted by subscripts 1 and 2 and the firms be denoted by subscripts X and Y. The utility functions for the two households are similar in function form but differ in their parameters: household i chooses its consumption of goods X and Y, and its consumption of leisure (J_i), to maximize the following:

$$u_i = C_{ix}^{\alpha_i} C_{iy}^{\beta_i} J_i^{1-\alpha_i-\beta_i}$$

Each household has an endowment of time, H_i , which it allocates between leisure and labor supply. The two households supply different kinds of labor (e.g., engineers vs. lawyers) and are therefore paid different wage rates (w_i). In addition, each household receives a lump-sum transfer, R_i , from the government.

The two firms have linear production functions and earn zero profits. Firm X uses only labor of type 1 and firm Y uses only labor of type 2:

$$Q_x = L_1 \text{ and } Q_y = L_2$$

The government imposes three ad valorem taxes: sales taxes on the two goods (T_x and T_y) and a tax on wages (T_w). It runs a balanced budget and distributes all tax revenue via the lump-sum transfers. Let p_x be the producer price of X, p_y be the producer price of Y, w_1 be the pre-tax wage paid to type-1 workers and w_2 be the pre-tax wage to type-2 workers. The budget constraint for household i is thus:

$$p_x(1+T_x)C_{ix} + p_y(1+T_y)C_{iy} = w_i(1-T_w)L_i + R_i$$

Finally, you may assume the government splits total revenue among the two households in fixed proportions: each receives half of total revenue (that is, it always chooses $R_1 = R_2$).

Please answer the following questions.

- (1) Derive household i 's demand equations, labor supply equation, indirect utility function, expenditure function and price index.
- (2) Using the parameter values and exogenous data in Table 1, construct and solve a numerical version of the model in Ox. In addition, carry out an appropriate homogeneity test on your model. Please report the results for each simulation.
- (3) Use your model to carry out the following policy experiments: an increase of 0.1 in (1) T_x , (2) T_y , and (3) T_w . For each experiment, report results for all variables, show the distributional effects across the two households, and calculate the marginal excess burden.

Table 1: Values of Parameters and Variables

Parameter or Variable	Value
α_1	.6
β_1	.2
H_1	100
α_2	.4
β_2	.4
H_2	100
T_x	0.1
T_y	0.2
T_w	0.1