

Solution to Exercise 1

1 Deriving Individual Demand and Supply Curves

Both curves are essentially given in the problem. The individual demand curve can be obtained from the willingness to pay equation by observing that the person will buy any pizzas for which her willingness to pay is greater than or equal to the price of a pizza. Her last, or marginal, pizza will therefore be the one for which $W2P = P$. Thus:

$$\begin{aligned} W2P_i &= 10 - Q_i && \text{Given in the problem} \\ W2P_i &= P && \text{For the last pizza demanded} \end{aligned}$$

Combining gives the individual demand curve, which can be expressed in two equivalent ways:

$$\begin{aligned} P &= 10 - Q_i \\ Q_i &= 10 - P \end{aligned}$$

The individual supply curve can be obtained in a similar fashion. Suppliers will sell any pizzas for which the price is greater than or equal to their marginal cost. The marginal pizza, therefore, will be the one where $MC = P$. Thus:

$$\begin{aligned} MC_j &= 2 * Q_j && \text{Given in the problem} \\ MC_j &= P && \text{For the last pizza supplied} \end{aligned}$$

Combining:

$$\begin{aligned} P &= 2 * Q_j \\ Q_j &= P/2 \end{aligned}$$

2 Deriving the Market Demand and Supply Curves

The market demand curve is obtained by adding up the quantities demanded by individuals at each price:

$$Q_M^D = \sum_{i=1}^{20} Q_i$$

Plugging in the demand equation from part 1:

$$Q_M^D = \sum_{i=1}^{20} (10 - P)$$

Since nothing inside the sum depends on i , the sum is easy to evaluate: it's just 20 times the individual demand:

$$Q_M^D = 20 * (10 - P)$$

$$Q_M^D = 200 - 20 * P$$

The same procedure can be applied to the supply side:

$$Q_M^S = \sum_{j=1}^{10} Q_j$$

$$Q_M^S = \sum_{j=1}^{10} (P / 2)$$

$$Q_M^S = 10 * (P / 2)$$

$$Q_M^S = 5 * P$$

3 Finding the Equilibrium Price and Quantity

Finding the equilibrium price and quantity is just a matter of finding the point where the demand and supply curves cross. In algebra, it amounts to solving the following three simultaneous equations:

$$Q_M^D = 200 - 20 * P$$

$$Q_M^S = 5 * P$$

$$Q_M^D = Q_M^S$$

Plugging the first two equations into the third eliminates everything but P:

$$200 - 20 * P = 5 * P$$

$$200 = 25 * P$$

$$P = \$8$$

Using this to find the number of pizzas demanded:

$$Q_M^D = 200 - 20 * 8$$

$$Q_M^D = 40$$

Check by calculating the number of pizzas supplied, which should be the same:

$$Q_M^S = 5 * 8$$

$$Q_M^S = 40$$

4 Analyzing the Outcome for Individual Buyers

To figure out what happens to an individual buyer, the best thing to do is to go back to the individual's demand equation and put in the market price:

$$Q_i = 10 - P$$

$$Q_i = 10 - 8$$

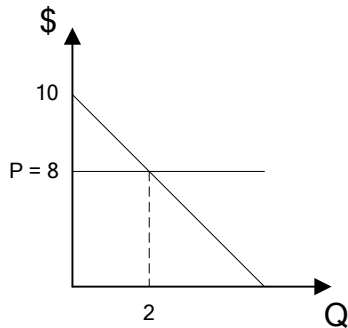
$$Q_i = 2$$

In this problem, it's possible to skip these steps and just divide the total number of pizzas, 40, by the number of individuals, 20, to get 2 pizzas per person. However, that approach only works when all of the buyers are identical. Going back to the demand equations will always work, even when the buyers have different demands.

For this problem, the amount of consumer surplus can be computed in two different ways. One approach is to compute the CS on each pizza:

Pizza	W2P	P	CS
1	9	8	1
2	8	8	0

This method works for problems involving small integers. Alternatively, one could compute the area between the demand curve and the price:



$$CS = (1/2) * (2) * (10 - 8)$$

$$CS = 2$$

The difference between the two approaches becomes small when Q is large. In this class, we will use the area approach (the second method) unless otherwise noted. It's easier to use and it picks up the fact that people can often consume fractions of a unit: half a pizza, for example.

5 Analyzing the Outcome for Individual Sellers

Inserting the price into the individual supply curve:

$$Q_j = P/2$$

$$Q_j = 8/2$$

$$Q_j = 4$$

As before, it would be possible to skip steps in this problem and get Q_j by dividing 40 by 10. However, that only works when all the firms are identical.

The firm's profit will be its revenue less its total cost:

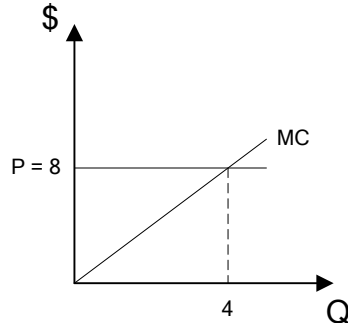
$$TR = \$8 * 4 = \$32$$

$$TC = 16 + Q^2 = \$32$$

Thus, profits are zero: the firms are exactly covering their total costs. Note that the firms DO get some producer surplus. Computing it by summing up PS on each pizza:

Pizza	P	MC	P
1	8	2	6
2	8	4	4
3	8	6	2
4	8	8	0

The total PS is thus \$12. Alternatively, PS can be computed as the area between the price line and the MC curve:



$$PS = (1/2) * (4) * (8-0)$$

$$PS = 16$$

Notice that PS is equal to the firm's fixed cost. That's not a coincidence. PS is always the difference between P and the firm's MC , so it's the amount by which revenue exceeds variable costs. When a firm is earning zero profit overall, as these firms are, its revenue must equal the sum of its variable and fixed costs. Thus, its revenue must exceed its variable costs by exactly its fixed costs.