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Economics 410

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Solution to Exercise 3

1 Market Demand and Equilibrium

The first step in finding the market demand curve is to solve the individual demand curve for Q_i as a function of *P*:

(1) $P = 200 - 300 * Q_i$

(2) $300 * Q_i = 200 - P$

(3) $Q_i = (200 - P)/300$

The next step is to add up over the town's population:

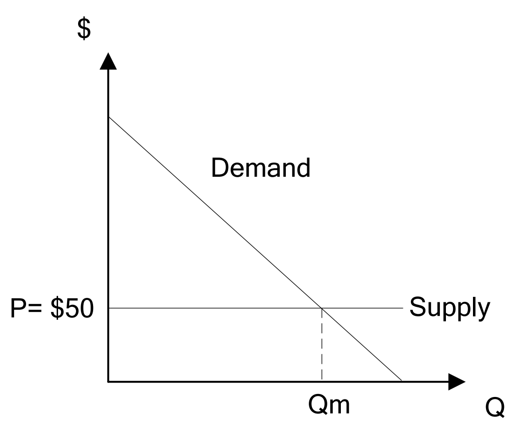
(4) $Q_m^d = \sum_{i=1}^{100k} Qi$

(5) $Q_m^d = 100{,}000 * Q_i$

(6) $Q_m^d = 100{,}000\left(\frac{200 - P}{300}\right)$

(7) $Q_m^d = \left(\frac{200{,}000 - 1{,}000 * P}{3}\right)$

Since the marginal cost is constant at $50, the supply curve is horizontal and the equilibrium price will be $50. The situation is shown in the diagram below.



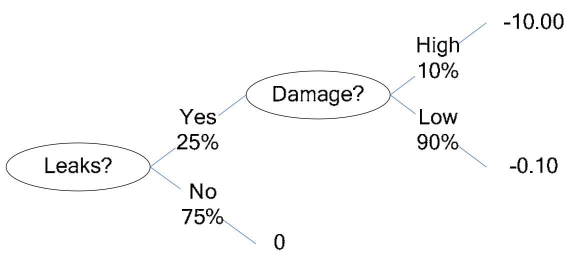
Solving for the equilibrium quantity is just a matter of inserting $50 for *P* in the demand equation:

(8) $$Q_m^d = \left(\frac{200{,}000 - 1{,}000 * 50}{3} \right)$$

(9) $$Q_m^d = \left(\frac{150{,}000}{3} \right) = 50{,}000$$

2 The Disposal Externality

There are multiple ways to approach this part of the problem but the easiest is probably to use a two step process. The first step is to calculate the expected annual damages from leakage. The corresponding part of the decision tree looks like this:



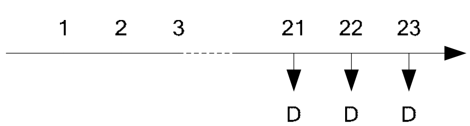
If the battery leaks, the EV of its damage is:

(10) 0.90*($0.10) + 0.10*($10.00) = $1.09

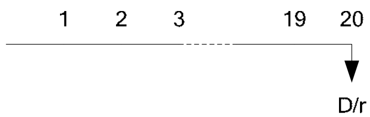
Moving one step to the left, the overall annual EV of damage from the battery is:

(11) 0.25*($1.09) + 0.75*($0) = $0.2725

The second step is to calculate the present value of the expected damages. The cash flow diagram looks like this, where *D* = $0.2725:



This is equivalent to a single payment of *D*/*r* in year 20:



Since the interest rate is 5%, the PV of the expected damages is thus:

(12) $$PV = \frac{(0.2725/0.05)}{(1.05)^{20}}$$

(13) PV = $2.054

In other words, every battery sold can be expected to cause damages equal to just over $2 in present value. That is, $2.054 is the marginal cost of the externality associated with the battery. It takes into account the long time horizon and all of the uncertainties associated with leakage.

3 The Efficient Outcome

To find the efficient outcome, the first step is to construct the marginal social cost curve:

(14) $$MSC = MC + MC_{EXT}$$

(15) MSC = $50 + $2.054 = $52.054

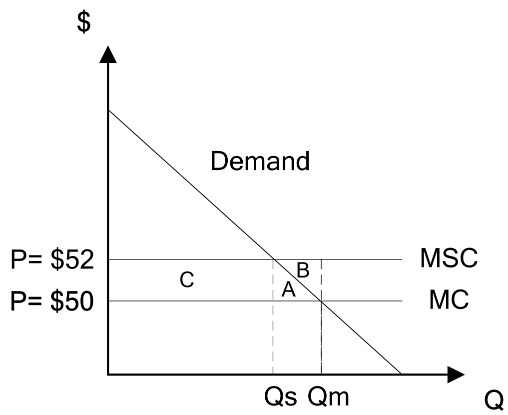
The efficient price should be equal to the *MSC*, so it should be $52.054 (rounding to $52 is fine but the numbers below would be slightly different). At this price, the quantity of batteries sold would be:

(16) $$Q_m^d = \left(\frac{200{,}000 - 1{,}000 * 52.054}{3} \right)$$

(17) $$Q_m^d = \left(\frac{147{,}946}{3} \right) = 49{,}315$$

To achieve this via a tax, the tax rate should be exactly equal to the externality cost: $2.054. Another way to see this is that the tax would need to be large enough to raise the price up to its efficient level. Since the efficient price is $52.054 and the original price is $50, the tax would have to be $2.054 per battery.

To calculate the effects of the tax on CS, government revenue and the externality problem, it helps to draw the following diagram:



Consumers lose CS equal to areas C+A; the government collects C in tax revenue (the tax times the number of batteries sold after the tax goes into effect, Qs); and the damage due to lead leaching is reduced by A+B (the externality times the reduction in batteries sold). The overall gain is area B:

Effect on consumers:	-(A+C)
Effect on the government:	+C
Change in externality:	+A+B
Net effect overall:	-(A+C)+C+A+B = B

To calculate the areas, the first step is to find the difference between Q_m and Q_s:

(18) $\Delta Q = Q_m - Q_s$

(19) $\Delta Q = 50{,}000 - 49{,}315 = 685$

Using this to calculate the areas:

(20) Area A = (1/2)*685*2.054 = $703

(21) Area B = (1/2)*685*2.054 = $703

(22) Area C = 49,315*2.054 = $101,293

The loss of CS is thus $101,293+$703 = $101,996, the change in government revenue is $101,293, the reduction in the externality is $703+$703 = $1406, and the overall welfare gain is $703.