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Economics 410

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Solution to Exercise 4

1 Original Market Equilibrium

Finding the market demand curve from the individual demand equation:

(1) $P = 100 - 4*Q_i$

(2) $4*Q_i = 100 - P$

(3) $Q_i = (100 - P)/4$

(4) $Q_m^d = \sum_{i=1}^{1M} Qi$

(5) $Q_m^d = 1{,}000{,}000 * Q_i$

(6) $Q_m^d = 1{,}000{,}000\left(\frac{100 - P}{4}\right)$

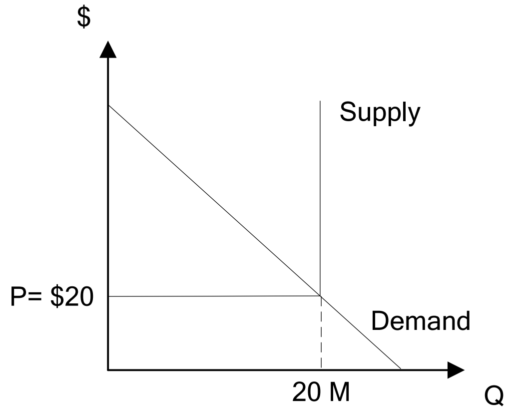
(7) $Q_m^d = 25{,}000{,}000 - 250{,}000 * P$

The market supply curve is a little bit more complicated than usual because of the capacity constraint on each firm. However, it's easy to figure out where it is by thinking about how much each firm would be willing to supply at various prices:

Price	Quantity Supplied
< \$20	0
\$20	Up to 200,000
> \$20	200,000

Since there are 100 identical firms, the market supply curve will be 100 times this: 0 when P is below \$20, up to 20 million when P = \$20, and exactly 20 million when P > \$20.

Given that the supply curve looks like that, the market equilibrium will either be: (1) on the horizontal portion and have P=\$20, (2) be on the vertical portion and have Q=20 million, or (3) be right at the corner with P=\$20 and Q=20 million. To find out which case applies, try them out in the demand equation. To test case 1, put P=\$20 into the demand equation to see if Q_m^d is less than or equal to 20 million. Since Q_m^d = 20 million at P=\$20, it's clear that case 3 actually applies: the equilibrium is at the corner:



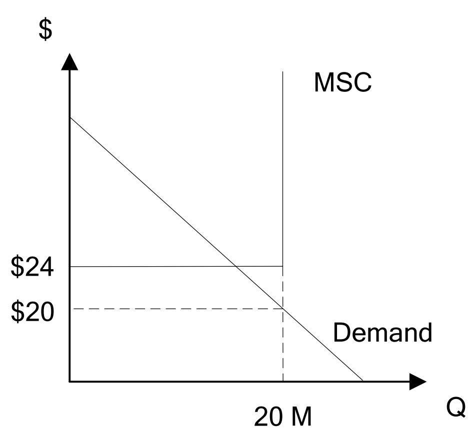
If Q_m^d had been lower than 20 million when P=$20, the equilibrium would have been on the horizontal portion of the supply curve with P=$20 and Q=$Q_m^d$. If Q_m^d had been higher than 20 million at P=$20, the equilibrium would have been on the vertical portion of the supply curve. The quantity would have been 20 million and the price could be found by putting Q_m^d=20 million into the demand curve and then solving for P.

At P=$20, each household's Q can be obtained from its demand curve:

(8) Q_i = (100 – 200)/4 = 20

That checks: scaling up to 1 million households gives a total demand of 20 million. Each firm produces at its 200K capacity. The equilibrium is not efficient: consumer and producer surplus on the 20 millionth unit are both 0 but production of that unit is causing $4 worth of environmental damage.

To find the efficient price and quantity, construct the marginal social cost curve by adding the $4 externality onto each firm's marginal cost curve. The result is a MSC that has the same basic shape as the original supply curve but is $4 higher:

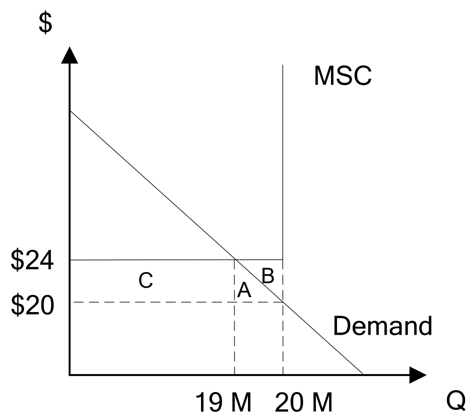


From the diagram, it's clear that the efficient price is $24. To find the efficient quantity, use the demand curve with P=$24:

(9) $Q_m^d = 25{,}000{,}000 - 250{,}000 * 24$

(10) $Q_m^d = 19{,}000{,}000$

The gain in social surplus will be area B in the diagram below:



Calculating:

(11) Area B = (1/2)(1M)*($4) = $2 million

In other words, the deadweight loss due to the externality is $2 million.

2 Equilibrium After the Population Increase

The first step is to find the new demand curve. The process is exactly the same except that now the population is twice as large:

(12) $Q_m^d = 2{,}000{,}000 * Q_i$

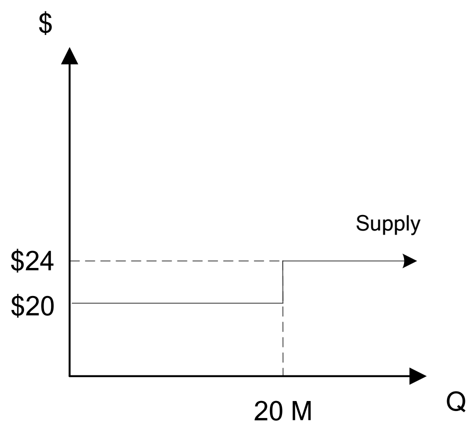
(13) $Q_m^d = 2{,}000{,}000\left(\frac{100 - P}{4}\right)$

(14) $Q_m^d = 50{,}000{,}000 - 500{,}000 * P$

The market supply curve is a little trickier because now there are two types of firms: 100 old firms, each with MC=$20 and a 200K capacity constraint, and an unknown number of new firms having MC=$24. However, finding the new supply curve is just a matter of applying the same reasoning used above but now keeping track of the two types of firms:

Price	Q Supplied by an Old Firm	Q Supplied by a New Firm
< $20	0	0
$20	Up to 200,000	0
$20<P<$24	200,000	0
$24	200,000	Any Amount

Graphically, the supply curve looks like this:



The market equilibrium will either: have P=$20 and Q<20 million (not likely since the demand curve has shifted out); have Q=20 million and a price between $20 and $24; or have P=$24 and Q>20 million. In the first two situations, only old firms will be in the market; in the last case, there will be a mix of old and new firms.

Using the demand curve to check the cases: if P=$20, Q_m^d = 40 million, so that can't be right (the most that would be supplied at P=$20 is 20 million). If Q_m^d = 20 million, P would have to be the solution to the following:

(15) $20{,}000{,}000 = 50{,}000{,}000 - 500{,}000 * P$

The value of P that satisfied the equation is $60, which is much higher than $24. That can't be an equilibrium because new firms would enter the market. The only remaining case is P=$24. Plugging that into the demand curve gives the market quantity: 38 million. That makes sense: it's twice as large as the quantity calculated above for P=$24.

The old firms earn a $4 profit (PS) on every unit they produce, so they'll supply 200,000 units each for a total of 20 million. The new firms will supply the remaining 18 million units.

3 Effect on Old Firms

The old firms are MUCH better off than they were in the original equilibrium. Each one is now earning $800,000 of producer surplus. At an interest rate of 5%, the present value of the stream would be \$800,000/0.05 = \$16 million. That \$16 million is the value of being grandfathered. To the group of old firms as a whole, grandfathering is worth \$1.6 billion. The political implication is that the old firms will have an *enormous* incentive to lobby for two things: (1) keeping the new regulation in place (because it raises P) and (2) preservation of their grandfathered status.