A Numerical Implementation of the Harberger Model

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This note describes the theory underlying the numerical version of the Harberger Model implmented in the accompanying Ox files.

1 Basic Structure

- Two sectors: X and Y
- Each sector uses capital and labor to produce its good according to a CRTS constant elasticity of substitution (CES) function. Sector X is capital intensive; sector Y is labor intensive.
- Four households: A, B, C, D. Households are differentiated by source of income and pattern of expenditure. A and B own labor and capital while C and D own only labor. Households A and C have identical preferences and spend most of their income on X; households B and D have identical preferences and spend most of their income on Y.
- Government: g
- Taxes on factors and goods

2 Sectors X and Y

Each has a CES unit cost function but parameters δ_j and σ_j differ:

$$c_j = \left(\delta_j r_j^{1-\sigma_j} + (1-\delta_j)w^{1-\sigma_j}\right)^{\frac{1}{1-\sigma_j}} \tag{1}$$

Cost of capital may vary by sector:

$$r_j = r + \tau_{kj}$$

Factor demand equations:

$$k_j = \delta_j \left(\frac{c_j}{r_j}\right)^{\sigma_j} q_j \tag{2}$$

$$l_j = (1-\delta_j)\left(\frac{c_j}{w}\right)^{\sigma_j} q_j \tag{3}$$

Cost shares:

$$\frac{r_j k_j}{c_j q_j} = \delta_j \left(\frac{c}{r_j}\right)^{\sigma-1} \tag{4}$$

$$\frac{w l_j}{c_j q_j} = \left(1-\delta_j\right)\left(\frac{c}{w}\right)^{\sigma-1} \tag{5}$$

3 Households and the Government

Both households and the government are represented by CES utility functions over goods X and Y. All have an identical elasticity of substitution but expenditure weights vary.

$$u_i = \left(\alpha_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha_i)^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{6}$$

Household budget constraint, $h \in \{A,B,C,D\}$:

$$m_h = rk_h + w(1-\tau_w)l_h$$

Government budget constraint:

$$m_g = \tau_{kx}k_x + \tau_{ky}k_y + \tau_w w \sum_i l_i + \tau_x q_x + \tau_y q_y$$

Demands for goods X and Y:

$$x_i = \frac{\alpha_i m_i}{p_{ci}}\left(\frac{p_{ci}}{p_x}\right)^{\sigma} \tag{7}$$

$$y_i = \frac{(1-\alpha_i)m_i}{p_{ci}}\left(\frac{p_{ci}}{p_y}\right)^{\sigma} \tag{8}$$

Indirect utility function for households:

$$v_i = m_i \left(\alpha_i p_x^{1-\sigma} + (1-\alpha_i) p_y^{1-\sigma}\right)^{\frac{1}{\sigma-1}} \tag{9}$$

Expenditure function:

$$e_i = u_i \left(\alpha_i p_x^{1-\sigma} + (1-\alpha_i) p_y^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{10}$$

Price index:

$$p_{ci} = \left(\alpha_i p_x^{1-\sigma} + (1-\alpha_i) p_y^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{11}$$

4 Prices and Markets

$$l_x + l_y = \sum_h l_h$$

$$k_x + k_y = \sum_h k_h$$

$$q_x = \sum_i x_i$$

$$q_y = \sum_i y_i$$

$$p_x = c_x + \tau_x$$

$$p_y = c_y + \tau_y$$