

Solution to Exercise 5

1 Market Demand Equation

The first step in finding the market demand curve is to determine the values of A and B in the individual demand equation:

$$(1) \quad P = A - B * Q_i$$

Since everyone has the same demand curve, the experimental study provides three data points that can be used to determine A and B:

Person	P	Q
1	\$1.00	18
2	\$0.50	19
3	\$1.50	17

Any two are sufficient to determine A and B. Using the data for person 1 and person 2, the process goes as follows:

$$(2) \quad \$1.00 = A - B * 18 \quad \text{Person 1}$$

$$(3) \quad \$0.50 = A - B * 19 \quad \text{Person 2}$$

Solving the first equation for A:

$$(4) \quad A = \$1.00 + B * 18$$

Inserting this into the second equation and solving for B:

$$(5) \quad \$0.50 = (\$1.00 + B * 18) - B * 19$$

$$(6) \quad -\$0.50 = -B$$

$$(7) \quad B = \$0.50$$

Inserting this into the equation for A:

$$(8) \quad A = \$1.00 + \$0.50 * 18 = \$10$$

Thus, the individual demand equation is:

$$(9) \quad P = \$10 - \$0.50 * Q_i$$

Rearranging to give Q as a function of P:

$$(10) \quad Q_i = (\$10 - P)/\$0.50$$

$$(11) \quad Q_i = 20 - 2P$$

Solving for the market demand:

$$(12) \quad Q_m^d = \sum_{i=1}^{10K} Q_i$$

$$(13) \quad Q_m^d = 10,000 * Q_i$$

$$(14) \quad Q_m^d = 10,000(20 - 2P)$$

$$(15) \quad Q_m^d = 200,000 - 20,000 * P$$

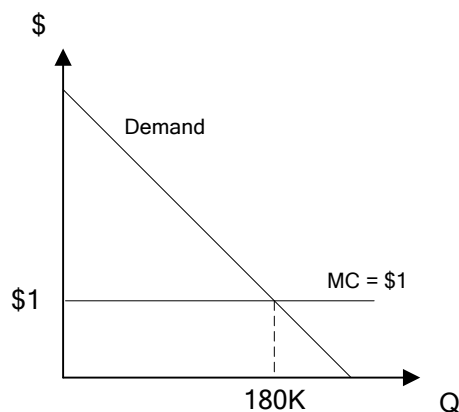
The market demand equation can also be rearranged and written this way:

$$(16) \quad P = 10 - (1/20,000)Q_m^d$$

This form is a little more convenient later in the problem.

2 Market Equilibrium

Finding the market equilibrium is straightforward since the MC is constant at \$1 per trip. Graphically, the situation looks like this:



The demand and supply intersect where $P = MC = \$1$. The quantity demanded at that point can be found by inserting \$1 into the market demand curve:

$$(17) \quad Q_m^d = 200,000 - 20,000 * 1 = 180,000$$

This quantity is efficient because the value of the last ride to the last rider (her W2P) is \$1, which is exactly equal to the cost of providing the trip. *All* rides worth more than \$1 are being produced (those to the left of 180,000), and *no* rides valued less than \$1 are being produced (those to the right of 180,000). There is no possible change in Q that would make someone better off without leaving someone else worse off.

Total consumer surplus is the area of the triangle above $P = \$1$:

$$(18) \quad CS = (1/2)*(\$10-\$1)*(180,000)$$

$$(19) \quad CS = \$810,000$$

Because $P = MC$, there is no producer surplus.

3 Inefficiency of the Market Equilibrium

The market equilibrium is inefficient because the cost of the last ride consumed is \$1 but the total social benefit of that ride is \$1.50 when the externality is included (\$1 to the rider plus \$0.50 of externality benefits). Since the benefits of the last ride are substantially higher than the cost of producing it, there would be net gains from increasing the number of trips. Therefore, the market outcome is not efficient when the externality is present.

[*Note: It's OK to do the following calculation in the answer to part 4 instead of here.*] At the efficient number of trips, the marginal social benefit of the last trip should be just equal to the cost of producing it. In algebra, the efficient Q is the one where the following is true:

$$(20) \quad MSB = MC$$

The first step in finding Q is to find an equation for MSB:

$$(21) \quad MSB = W2P + MB_{ext}$$

$$(22) \quad MSB = 10 - (1/20,000)Q_m^d + \$0.50$$

$$(23) \quad MSB = 10.50 - (1/20,000)Q_m^d$$

Now find the Q where MSB is equal to MC:

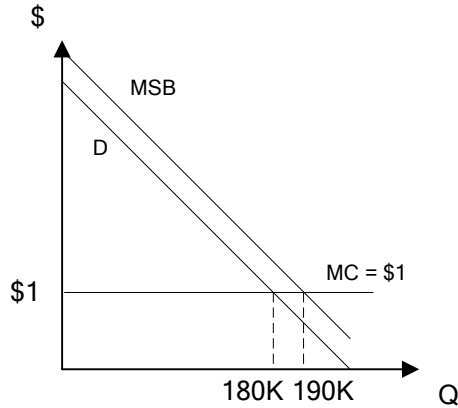
$$(24) \quad MSB = MC$$

$$(25) \quad 10.50 - (1/20,000)Q_m^d = \$1$$

$$(26) \quad (1/20,000)Q_m^d = 9.50$$

$$(27) \quad Q_m^d = 190,000$$

Thus, the market produces 10,000 too few bus trips. Graphing the situation:



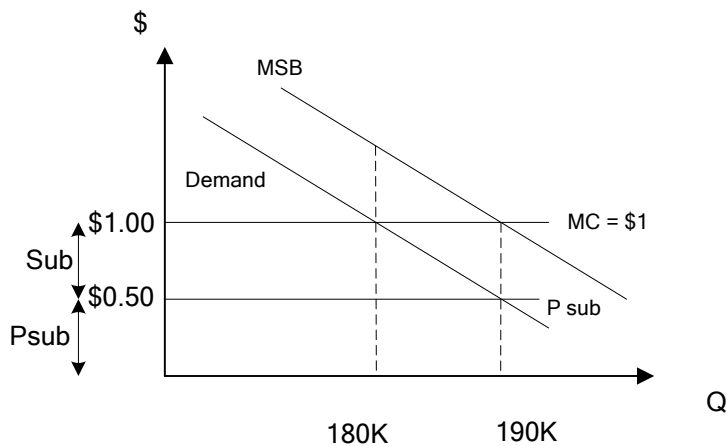
4 Efficient Subsidy

To reach the efficient number of rides, the subsidy needs to lower the price by enough that riders will demand 190,000 rides. We can find the subsidized price (P_{sub}) by using the market demand curve with the quantity set to 190,000:

$$(28) \quad P_{sub} = 10 - (1/20,000)(190,000)$$

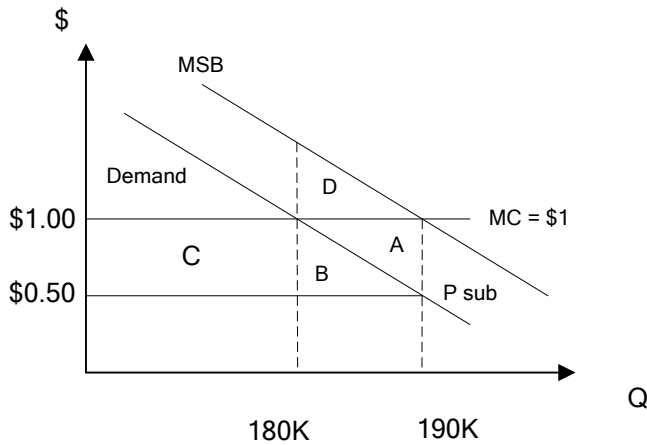
$$(29) \quad P_{sub} = 10 - 9.50 = 0.50$$

In other words, the subsidy must be large enough to lower the price to \$0.50. Since the marginal cost is \$1, the subsidy must be \$0.50 in order to bring the price down to \$0.50 ($P_{sub} = MC - \text{Subsidy}$). Graphing:



5 Distributional Effects

The distributional effects of the policy are shown by the labeled areas in the graph below:



The total value of the subsidy is the subsidy per ride (\$0.50) times the number of rides taken once the subsidy is in place (190,000). It's equal to the sum of areas C, B, and A and is \$95,000. Of this, area C (\$90,000) is a transfer to people who would have ridden the bus anyway, and area B (\$2,500) is new consumer surplus to people who now take the bus but didn't before the subsidy. The value of positive externalities created by the additional bus riders is $\$0.50 \times 10,000$ (\$5,000), which corresponds to areas D+A on the graph. The net gain to the economy as a whole is thus $-(\text{subsidy: } C+B+A) + (\text{CS: } C+B) + (\text{Extern: } A+D) = \text{area D}$ (\$2,500). Area A, by the way, is part of the subsidy but is not part of the CS received by riders – it covers the extra costs imposed by the new riders above and beyond their own willingness to pay.