

## Solution to Exercise 6

### 1 Finding the Efficient Amounts of Abatement

There are two ways to find the efficient amount of abatement for each plant. Both methods are correct and you can use either one.

#### *Method 1*

The efficient amount of abatement is the point at which  $MBA = MCA$ . Since each ton of emissions cause \$50,000 of damage, the marginal benefit of abatement is \$50,000 (the damages avoided). Each plant should clean up its emissions until its MCA is equal to \$50,000.

Plant 1:

$$(1) \quad 50,000 = 1,000 Q_1$$

$$(2) \quad 50 = Q_1$$

Plant 2:

$$(3) \quad 50,000 = 10,000 Q_2$$

$$(4) \quad 5 = Q_2$$

Plant 3:

$$(5) \quad 50,000 = (10,000/3) Q_3$$

$$(6) \quad 15 = Q_3$$

Total abatement:  $50+5+15 = 70$  tons.

#### *Method 2*

An alternate way to find the answer is to start by constructing the overall MCA curve by combining the MCA curves for the individual plants. This approach involves a few more steps but it's more versatile than the first method, which only works when MBA is equal to a constant. Here is how it goes.

For efficiency,  $Q_1$ ,  $Q_2$  and  $Q_3$  must be set so that  $MCA_1 = MCA_2 = MCA_3$ . Since the MCA's will be equal we can drop the subscripts and just call them "MCA". Putting this into the MCA equations:

$$(7) \quad \text{Plant 1:} \quad MCA = 1,000 Q_1$$

(8) Plant 2:  $MCA = 10,000 Q_2$

(9) Plant 3:  $MCA = (10,000/3) Q_3$

Solving each equation for Q in terms of MCA:

(10) Plant 1:  $Q_1 = MCA / 1,000$

(11) Plant 2:  $Q_2 = MCA / 10,000$

(12) Plant 3:  $Q_3 = 3 * MCA / 10,000$

Summing to get the total amount of abatement for a given MCA:

(13)  $Q_T = Q_1 + Q_2 + Q_3$

(14)  $Q_T = MCA / 1,000 + MCA / 10,000 + 3 MCA / 10,000$

(15)  $Q_T = ( 1/1,000 + 1/10,000 + 3/10,000 ) MCA$

(16)  $Q_T = ( 14/10,000 ) MCA$

Rearranging produces the overall MCA:

(17)  $MCA = (10,000/14) Q_T$

The efficient level of  $Q_T$  is the one where  $MBA = MCA$ :

(18)  $MBA = MCA$

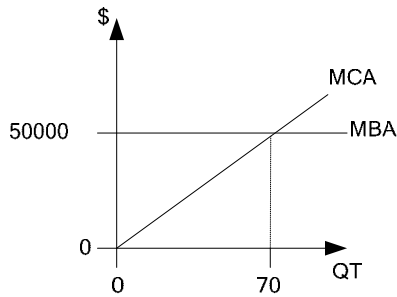
(19)  $50,000 = (10,000/14) Q_T$

(20)  $70 = Q_T$

Expressed in terms of the original 350 tons of emissions, the efficient reduction is thus:

(21)  $70/350 = 0.20 = 20\%$

Graphically, the situation looks as follows:



The  $MBA = MCA$  step is what makes method 2 more versatile than method 1: it could be used even when  $MBA$  is not constant.

*Example Problem with Sloping MBA*

This section is an illustration only – it is not part of the assignment.

Suppose that the problem had specified that the  $MBA$  was given by  $MBA = 70,000 - 700 QT$ . Since  $QT$  is initially unknown, it's not possible to say what the efficient  $MBA$  is right off the bat and method A can't be used. However, the only change to method B is in the  $MBA = MCA$  step:

(22)  $MBA = MCA$

(23)  $70,000 - 700 QT = (10,000/14) QT$

(24)  $98 - 0.98 QT = QT$

(25)  $98 = 1.98 QT$

(26)  $49.5 = QT$

Inserting this into the  $MBA$  and  $MCA$  curves to determine  $MBA$  and  $MCA$  at the efficient point:

(27)  $MBA = 70,000 - 700*49.5 = 35,353.5$

(28)  $MCA = (10,000/14)*49.5 = 35,353.5$

Once these values are known, the reduction can be allocated across plants using the steps in method A: set each plant's  $MCA$  equal to 35,353.5.

*End of Example*

Shifting back to the actual problem in the exercise, the remaining step in method 2 is to figure out how the 70 ton reduction should be allocated across plants. That can be done by following the simultaneous equations approach used in the example in class:

$$(29) \quad MCA1 = MCA2$$

$$(30) \quad MCA2 = MCA3$$

$$(31) \quad Q1 + Q2 + Q3 = 70$$

The results will be the same as the numbers calculated earlier.

*End of Method 2*

Finally, under either approach the total cost of abatement is the sum of what each plant spends cleaning up (the area under its MCA curve):

$$(32) \quad \text{Plant 1:} \quad (1/2) * 50 * 50,000 = 1.25 \text{ million}$$

$$(33) \quad \text{Plant 2:} \quad (1/2) * 5 * 50,000 = 125,000$$

$$(34) \quad \text{Plant 3:} \quad (1/2) * 15 * 50,000 = 375,000$$

Total: \$1.75 million

## **2 Equal Percentage Reductions**

If each plant reduced its emissions by 20 percent, emissions would still drop by 70 tons but the overall cost would be higher. Calculating each plant's Q of abatement:

$$(35) \quad \text{Plant 1:} \quad Q1 = 0.2 * 100 = 20$$

$$(36) \quad \text{Plant 2:} \quad Q2 = 0.2 * 50 = 10$$

$$(37) \quad \text{Plant 3:} \quad Q3 = 0.2 * 200 = 40$$

The corresponding marginal costs:

$$(38) \quad \text{Plant 1:} \quad MCA1 = 1,000 * 20 = \$20,000$$

$$(39) \quad \text{Plant 2:} \quad MCA2 = 10,000 * 10 = \$100,000$$

$$(40) \quad \text{Plant 3:} \quad MCA3 = (10,000/3) * 40 = \$133,333.33$$

Areas under the MCA curves:

(41) Plant 1:  $(1/2) * 20 * \$20,000 = \$200,000$

(42) Plant 2:  $(1/2) * 10 * \$100,000 = \$500,000$

(43) Plant 3:  $(1/2) * 40 * \$133,333 = \$2,666,667$

Total: \$3.367 million

An equal reduction policy costs about twice as much as an efficient approach.

### 3 Emissions Tax

Under an emissions tax, each plant would pay for abatement up to the point where its MCA equaled the tax. To achieve the efficient amount of abatement, the tax should be set to the value of MBA, which is \$50,000. (In general, to get the right amount of abatement, an emissions tax should be set to the dollar value of MBA at the intersection of the MCA and MBA curves.) The cost to each firm will be the sum of what it pays in abatement (from part 1) and what it pays in taxes (\$50,000 times its remaining emissions). The results are summarized in the table below:

| Plant | Original Emissions | Amount of Abatement | Remaining Emissions | Abatement Cost | Taxes  | Total Cost |
|-------|--------------------|---------------------|---------------------|----------------|--------|------------|
| 1     | 100                | 50                  | 50                  | 1.25 M         | 2.5 M  | 3.75 M     |
| 2     | 50                 | 5                   | 45                  | 125 K          | 2.25 M | 2.375 M    |
| 3     | 200                | 15                  | 185                 | 375 K          | 9.25 M | 9.625 M    |
| Total | 350                | 70                  | 280                 | 1.75 M         | 14 M   | 15.75 M    |

### 4 Comparing Policies

Each of the plants would prefer the inefficient approach from part 2 because it imposes lower total costs when tax payments are taken into account. From an economic perspective, however, the part 2 approach is bad because it wastes resources: the actual cost of cleaning up pollution is twice what it should be: \$3.367 million instead of \$1.75 million.

### 5 Tradable Permit System

A permit system should issue a total of 280 permits: enough for the emissions remaining after 70 tons have been cleaned up.

No matter how the permits are initially distributed, the firms will trade them until their marginal abatement costs are equal:  $MCA_1 = MCA_2 = MCA_3$ . From that we know that the permits will eventually end up distributed like the efficient emissions: 50 at plant 1, 45 at plant 2, and 185 at

plant 3. As discussed in class, the equilibrium price of a permit will be equal to the efficient MCA (that is, it will equal  $MCA1 = MCA2 = MCA3$ ), which is \$50,000.

To equalize overall costs, we can tweak the distribution of permits so that firms that need to pay less overall will have permits to sell (plant 1), and firms that need to pay more overall will have to buy permits (plants 2 and 3).

The table below summarizes the abatement costs and equalized costs across the three plants. Note: it's OK to round the equalized costs rather than carrying around the fraction. If you do that, it is a good idea to adjust the rounded numbers so that the total comes out to 1.75M; for example, it would be OK to round them to 583, 583 and 584.

| Plant | Amount of Abatement | Abatement Cost | Equalized Cost | Difference  |
|-------|---------------------|----------------|----------------|-------------|
| 1     | 50                  | 1.25 M         | 583 1/3 K      | - 666 2/3 K |
| 2     | 5                   | 125 K          | 583 1/3 K      | + 458 1/3 K |
| 3     | 15                  | 375 K          | 583 1/3 K      | + 208 1/3 K |
| Total | 70                  | 1.75 M         | 1.75 M         | 0           |

To lower plant 1's costs by \$666 2/3 K, it needs to be able to sell  $\$666\ 2/3\ K / \$50K = 13\ 1/3$  permits; that is, selling 13 1/3 permits at \$50,000 each brings in \$666 2/3 K of revenue. Plants 2 and 3, on the other hand, need to pay more and will have to buy permits. Plant 2 will need to be given  $\$458\ 1/3\ K / \$50K$ , or 9 1/6 permits less than it really needs. Similarly, plant 3 should be given 4 1/6 permits less than it needs. The results are summarized below:

| Plant | Efficient Emissions | <i>Plus</i><br>Permits to Sell | <i>Less</i><br>Permits to Buy | <i>Equals</i><br>Permits Allocated |
|-------|---------------------|--------------------------------|-------------------------------|------------------------------------|
| 1     | 50                  | 13 1/3                         | 0                             | 63 1/3                             |
| 2     | 45                  | 0                              | 9 1/6                         | 35 5/6                             |
| 3     | 185                 | 0                              | 4 1/6                         | 180 5/6                            |
| Total | 280                 | 13 1/3                         | 13 1/3                        | 280                                |