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Economics 410

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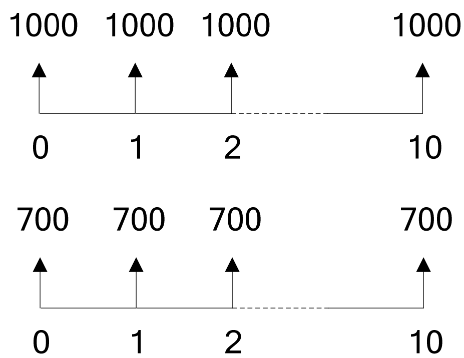
Solution to Exercise 2

There are two equally good ways to set up the problem. The notes below describe the two methods and the last page shows a spreadsheet that implements the second approach.

Approach 1: Gross Values in Each Use

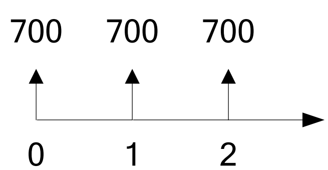
One approach is to compute the gross PV of the forest in each of its alternative uses: logging and no logging. If the forest is logged when the external benefit is $700, it will produce the following cash flows, where the profits from logging and the externalities are shown separately:

Cash flows if used for logging:



Equivalently, these could be combined into a single set of 11 cash flows of $1700. If the forest is not logged, it will produce the following:

Cash flows if never logged:

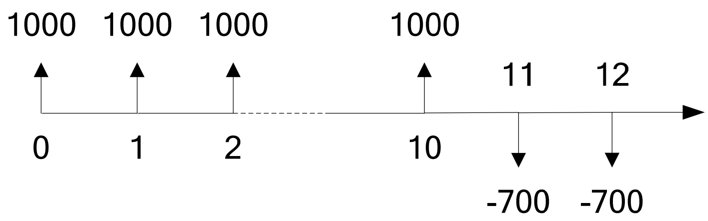


The NPV of logging is the PV of the first group of cash flows ($1000 + $700 for years 0-10) less the PV of the second group ($700 from 0 to infinity).

Approach 2: Net Value of Switching to Logging

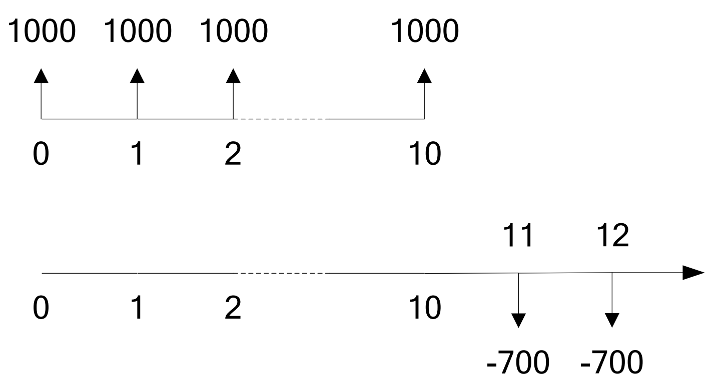
The second approach is to treat leaving the forest alone as "business-as-usual" and compute the NPV of switching to logging. In that case, there will be a net gain of $1000 during years 0 to 10 (since logging doesn't eliminate the externalities during that period) and a net loss of $700 in years 11 to infinity (since the forest is now producing 0 externalities and it would have produced 700 in the business-as-usual case). The cash flow diagram looks like this:

Net cash flows from switching to logging:



This is exactly the same as the first case except that the difference between logging and not logging has been computed year by year before taking the PV (that is, it's the PV of the differences). Under approach 1, the PV is calculated first and then the subtraction is done (that is, the difference of the PVs) The two approaches are mathematically equivalent and you can use either one.

The spreadsheet on the next page shows how to implement approach 2. It splits the cash flows up as follows to make the calculation more convenient:



It then the converts the second stream into an equivalent lump sum payment in year 10 using the B/r formula.

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Present Value of Logging

The cash flows in this section show the net gain during years 0 to 10 from allowing logging.

Year	Payment	Interest Rate 4%	5%	6%	
0	1000	1000	1000	1000	<-- Each cell is pmt/(1+r)^year
1	1000	962	952	943	
2	1000	925	907	890	
3	1000	889	864	840	
4	1000	855	823	792	
5	1000	822	784	747	
6	1000	790	746	705	
7	1000	760	711	665	
8	1000	731	677	627	
9	1000	703	645	592	
10	1000	676	614	558	
	Subtotal:	9111	8722	8360	<-- Total PV for each interest rate

Year 10 Total Present Value of Lost Externality Benefits

If the forest is logged, all of the externality benefits from year 11 on will be lost. Compute the PV of that stream as of year 10.

Annual Value	Interest Rate 4%	5%	6%	
-600	-15000	-12000	-10000	<-- Each is value/r
-700	-17500	-14000	-11667	
-800	-20000	-16000	-13333	

Year 0 Total Present Value of Lost Amenity

Now convert the year-10 value of the amenity to year 0.

Annual Value	Interest Rate 4%	5%	6%	
-600	-10133	-7367	-5584	<-- Each is (value/r)/(1+r)^10
-700	-11822	-8595	-6515	
-800	-13511	-9823	-7445	

Net PV of Logging Under Each Contingency

Construct a summary table.

Annual Value	Interest Rate 4%	5%	6%
-600	-1023	1355	2776
-700	-2711	127	1845
-800	-4400	-1101	915

Proceed with logging in any of these cases. Don't log otherwise. Put the corresponding amount from the table below into an escrow account in year 0. At year 11, begin paying compensation equal to the lost external benefits.

Escrow Account in Year 0

Annual Value	Interest Rate 4%	5%	6%
-600		7367	5584
-700		8595	6515
-800			7445

These would grow into the year-10 value of the lost amenity. In year 11 and beyond, they would generate exactly enough interest to compensate for the lost benefits.