

Solution to Exercise 1

1 Deriving Individual Demand and Supply Curves

Demand equation for a type-A person:

- $WTP_{ai} = 100 - Q_{ai}$ *Given*
- $WTP_{ai} = P^d$ *For the last unit demanded*
- $P^d = 100 - Q_{ai}$ *Combining*
- $Q_{ai} = 100 - P^d$ *Solving for Q_{ai}*

Demand equation for a type-B person:

- $WTP_{bi} = 100 - 2 \cdot Q_{bi}$
- $P^d = 100 - 2 \cdot Q_{bi}$
- $Q_{bi} = (100 - P^d) / 2$

Supply equation for a type-X firm:

- $WTA_{xi} = 10 + (1/50) \cdot Q_{xi}$ *Given*
- $WTA_{xi} = P^s$ *For the last unit sold*
- $P^s = 10 + (1/50) \cdot Q_{xi}$ *Combining*
- $Q_{xi} = 50 \cdot (P^s - 10)$ *Solving for Q_{xi}*

In passing, it's worth noting that the type-X firms will produce 0 when $P^s < 10$. That is, the type-X willingness to accept curve starts at 10 when $Q_{xi} = 0$ and goes up from there.

Supply equation for a type-Y firm:

- $WTA_{yi} = Q_{yi}$
- $Q_{yi} = P^s$

2 Deriving the Market Demand and Supply Curves

The market demand curve is obtained by adding up the quantities demanded by all of the individuals:

- $Q^D = \sum_{i=1}^{100} Q_{ai} + \sum_{i=1}^{50} Q_{bi}$
- $Q^D = 100 \cdot Q_{ai} + 50 \cdot Q_{bi}$
- $Q^D = 100 \cdot (100 - P^d) + 50 \cdot (100 - P^d) / 2$
- $Q^D = 12,500 - 125 \cdot P^d$

Going through the same steps on the supply side:

- $Q^S = \sum_{i=1}^{16} Q_{xi} + \sum_{i=1}^{100} Q_{yi}$
- $Q^S = 16 \cdot Q_{xi} + 100 \cdot Q_{yi}$

To get the supply curve exactly right, it's necessary to take into account the behavior noted above of the type-X firms when $P_s < 10$. Below \$10, the market supply is just the sum of the output of the type-Y firms:

- $Q^S = 16 \cdot 0 + 100 \cdot Q_{yi}$
- $Q^S = 100 \cdot P^s$ *if* $P^s \leq 10$

When $P^s \geq 10$, the type-X firms will be in the market and the supply will be:

- $Q^S = 16 \cdot 50 \cdot (P^s - 10) + 100 \cdot Q_{yi}$
- $Q^S = 900 \cdot P^s - 8000$ *if* $P^s \geq 10$

3 Finding the Equilibrium Price and Quantity

Finding the equilibrium assuming that $P^s \geq 10$ (will check this assumption later):

- $Q^D = 12,500 - 125 \cdot P^d$
- $Q^S = 900 \cdot P^s - 8000$
- $Q^D = Q^S$

- $P^d = P^s$

No taxes or subsidies yet

Replacing P^d and P^s by P and going through the algebra:

- $Q^D = Q^S$
- $12,500 - 125 \cdot P = 900 \cdot P - 8,000$
- $20,500 = 1,025 \cdot P$
- $P = 20$

The assumption was correct: the price is indeed above \$10 and both types of firms will produce. Had the price turned out to be lower than \$10, it would be necessary to redo the last couple of steps using the supply curve with only the type-Y firms ($Q^S = 100 \cdot P^s$)

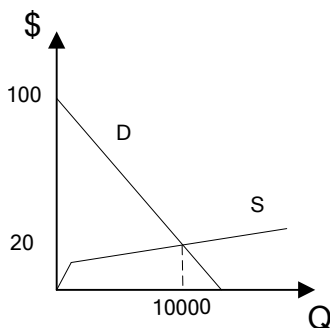
Solving for the quantity supplied:

- $Q^S = 900 \cdot 20 - 8,000$
- $Q^S = 10,000$

Check by solving for the quantity demanded:

- $Q^D = 12,500 - 125 \cdot 20$
- $Q^D = 10,000$

Graphing:



The kink in the supply curve is at $P = 10$, which is the point at which the type-X firms come on line.

4 Analyzing the Outcome for Buyers

Outcome for a type-A buyer and the CS for the group:

- $Q_{ai} = 100 - P^d = 100 - 20 = 80$
- $CS_{ai} = 0.5 \cdot (100 - 20) \cdot 80 = \$3,200$
- $CS_a = 100 \cdot \$3200 = \$320,000$

Outcome for a type-B buyer and the CS for the group:

- $Q_{bi} = (100 - P^d) / 2 = (100 - 20) / 2 = 40$
- $CS_{bi} = 0.5 \cdot (100 - 20) \cdot 40 = \$1,600$
- $CS_b = 50 \cdot \$1600 = \$80,000$

Total CS for all buyers:

- $CS = \$400,000$

5 Analyzing the Outcome for Sellers

Outcome for a type-X seller and the PS for the group (note that this accounts for the non-zero intercept in the type-X seller's *WTA* curve):

- $Q_{xi} = 50 \cdot (P^s - 10) = 500$
- $PS_{xi} = 0.5 \cdot (20 - 10) \cdot 500 = \$2,500$
- $PS_x = 16 \cdot \$2500 = \$40,000$

Outcome for a type-Y seller and the PS for the group:

- $Q_{yi} = P^s = 20$
- $PS_{yi} = 0.5 \cdot (20 - 0) \cdot 20 = \200
- $PS_y = 100 \cdot \$200 = \$20,000$

Total PS for all sellers:

- $PS = \$60,000$

Total SS (not required but convenient later):

- $SS = \$460,000$

6 Equilibrium with a Tax

The steps are similar to part 3 except that now there's a difference between P^d and P^s :

- $Q^D = 12,500 - 125 \cdot P^d$
- $Q^S = 900 \cdot P^s - 8,000$
- $Q^D = Q^S$
- $P^d = P^s + T$ *\$10.25 will be inserted later*
- $12,500 - 125 \cdot P^d = 900 \cdot P^s - 8,000$
- $12,500 - 125 \cdot (P^s + T) = 900 \cdot P^s - 8,000$
- $20,500 - 125 \cdot T = 1025 \cdot P^s$
- $19,218.75 = 1025 \cdot P^s$
- $P^s = 18.75$
- $P^d = 29$

Solving for the quantity using both the supply and demand equations (the second as a check):

- $Q^S = 900 \cdot P^s - 8,000 = 8,875$
- $Q^D = 12,500 - 125 \cdot P^d = 8,875$

7 Effect of the Tax on Social Surplus

New outcome for a type-A buyer and the CS for the group and change from the previous equilibrium:

- $Q_{ai} = 100 - P^d = 100 - 29 = 71$
- $CS_{ai} = 0.5 \cdot (100 - 29) \cdot 71 = \$2,520.25$
- $CS_a = 100 \cdot \$2,520.25 = \$252,050$

- $\Delta CS_a = \$252,050 - \$320,000 = -\$67,950$

New outcome for a type-B buyers:

- $Q_{bi} = (100 - P^d) / 2 = (100 - 29) / 2 = 35.5$

- $CS_{bi} = 0.5 \cdot (100 - 29) \cdot 35.5 = \$1,260.25$

- $CS_b = 50 \cdot \$1,260.25 = \$63,012.5$

- $\Delta CS_b = \$63,012.5 - \$80,000 = -\$16,987.5$

Total CS for all buyers:

- $CS = \$315,062.5$

- $\Delta CS = \$315,062.5 - \$400,000 = -\$84,937.5$

New outcome for type-X sellers:

- $Q_{xi} = 50 \cdot (P^s - 10) = 437.5$

- $PS_{xi} = 0.5 \cdot (18.75 - 10) \cdot 437.5 = \$1,914$

- $PS_x = 16 \cdot \$1,914 = \$30,625$

- $\Delta PS_x = \$30,625 - \$40,000 = -9,375$

Type-Y sellers:

- $Q_{yi} = P^s = 18.75$

- $PS_{yi} = 0.5 \cdot (18.75 - 0) \cdot 18.75 = \176

- $PS_y = 100 \cdot \$176 = \$17,578$

- $\Delta PS_y = \$17,578 - \$20,000 = -\$2,422$

Total PS for all sellers:

- $PS = \$48,203$

- $\Delta PS = \$48,203 - \$60,000 = -\$11,797$

Government revenue:

- $Rev = \$10.25 \cdot 8,875 = \$90,968.75$

New social surplus:

- $SS = \$315,062.5 + \$48,203 + \$90,968.75 = \$454,234.25$

Deadweight loss (calculated in two equivalent ways):

- $DWL = \$460,000 - \$454,234.25 = \$5,765$

- $DWL = 0.5 \cdot 10.25 \cdot (10,000 - 8,875) = \$5,765$

It's not required by the problem but it's interesting to calculate the DWL per dollar of revenue:

- $\$5,765 / \$90,969 = 0.063$

That is, each dollar of revenue causes about 6.3 cents of DWL.