Problem Set 1

(Due 9/10)

1. Normal Preferences

Ralph consumes only beer and pizza, and he likes both goods. Please answer the following questions about his preferences.

(a) Suppose that Ralph becomes sated with beer when his consumption is 8 cans and that beyond that he regards more beer as bad. Sketch some of his indifference curves.

(b) If Ralph is currently consuming 6 cans of beer and 3 pizzas and he is willing to give up two cans of beer to get an additional pizza, what is his marginal rate of substitution?

(c) Now suppose Ralph is never sated but he only consumes pizza and beer together: he drinks three cans of beer with each pizza. What might his indifference curves look like? Briefly explain why the curves look the way you drew them.

2. Unusual Preferences

Max Macho hates both quiche and white wine. The more he consumes of either, the more unhappy he becomes. He does, however, have convex preferences.

(a) Sketch an indifference curve for Max showing his preferences between slices of quiche and glasses of white wine. Show the direction in which he becomes better off.

(b) Do Max's indifference curves get flatter or steeper from left to right? Why?

3. Budget Constraints

Ivan consumes only vodka and caviar. Suppose the price of a shot of vodka is 10 roubles, the price of a dollop caviar is 20 roubles, and Ivan has an income of 80 roubles.

(a) Draw Ivan's budget constraint. Show which bundles are in his feasible set and which are not. Briefly discuss what the slope and intercepts of the budget constraint mean.

(b) Suppose that Ivan must wait an hour in line for each shot of vodka or dollop of caviar he buys. If Ivan has up to 4 hours to spend in line, show his feasible set.

(c) Now suppose that the government imposes a 100% tax on all vodka sold so the price to Ivan rises to 20 roubles per shot. Show how Ivan's budget constraint changes. Discuss what happened to his feasible set. Is he likely to care that the price has risen?

Problem Set 3

(Due 9/24)

1. Utility Maximization

Todd Turbo is about to buy a new car. Todd only cares about one thing in cars: top speed. However, he has to choose between cars with different engine sizes and numbers of gears. Suppose that top speed depends on the square root of the car's engine size multiplied by the car's number of gears. Let x be the size of the car's engine and let y be its number of gears.

(a) Explain why the utility function $U(x, y) = x^{1/2}y$ could be used to represent Todd's preferences.

(b) Suppose Todd's budget is \$1800, the price of a unit of engine size (say 1 cylinder) is \$100 and the price of each gear is \$240. How many cylinders and gears would Todd's optimal car have? Show this optimum graphically.

(c) Suppose Todd can't find any cars with more than 4 gears. Graph his optimum in this situation, being careful to show the slopes of all curves as accurately as possible. Calculate his marginal rate of substitution at that point and explain how it relates to the slope of his budget constraint.

2. Demand Functions and Engel Curves

Doug Cobblas has a utility function of the form $U(x, y) = x^{\alpha}y^{(1-\alpha)}$, where α is a constant. His total income is I, which he spends entirely on x and y.

(a) Derive Doug's demand function for x in terms of his income and the prices of the two goods.

(b) Using the results from part (a), graph Doug's demand for x when his income is \$100 and $\alpha = 0.2$.

(c) What is Doug's price elasticity of demand for x? Be sure to show how you obtained your answer.

(d) Assuming that $\alpha = 0.2$ and the price of x is \$2, graph Doug's Engle curve.

Problem Set 4

(Due 10/1)

1. Demand Elasticities

Lenny Lodgelizard likes to go on ski trips. On any particular trip, however, he is always torn between his two favorite activities: skiing and sitting around in the lodge bar talking about skiing. If he wants to ski, he has to buy a lift ticket but if he wants to sit in the bar he can buy drinks instead.

(a) Given the following information, calculate Lenny's price elasticity of demand for lift tickets, his cross price elasticity between drinks and lift tickets and his income elasticity of demand for lift tickets.

Ticket Price	Drink Price	Income	Tickets Bought
$25	$4	$20000	10
$29	$5	$20000	10
$29	$4	$20000	8
$25	$4	$25000	20

(b) Suppose that last year Lenny bought 10 lift tickets. Since then lift ticket prices have risen by 20%, the cost of a drink has gone up by 10% and Lenny's income has risen by 10%. How many lift tickets is Lenny likely to buy this year? Show how you got your answer.

(c) Are lift tickets and drinks in the bar complements or substitutes? How can you tell? Suppose that social convention changes and it becomes necessary to have a valid lift ticket on your ski jacket if you want to do any bragging in the bar. Are any of Lenny's elasticities likely to change? If so, describe the change in qualitative terms.

Problem 2 is on the back.

2. Income and Substitution Effects

Ronald McWendys only buys hamburgers and french fries together: exactly one bag of fries with each burger. Initially, the price of hamburgers is \$2, the price of a bag of fries is \$1 and Ronald spends \$12 a week on hamburgers and fries together.

(a) Graph Ronald's initial equilibrium.

(b) Now suppose the price of fries rises to \$2 per bag. Graph Ronald's new equilibrium and decompose the change into income and substitution effects. Explain anything unusual in your diagram.

(c) Calculate the compensating and equivalent variations for the price increase in part (b). Explain how you got your answer.

(d) Gus Glutton also eats hamburgers and french fries, but he only cares about the total caloric value of his food. Suppose that initially the price of a burger is \$2, the price of a bag of fries is \$1 and Gus buys 3 burgers and 7 bags of fries. For this problem, assume one hamburger has 400 calories and a bag of fries has 200. Graph Gus's initial equilibrium.

(e) Now suppose that the price of fries rises to \$2 per bag. Graph Gus's final equilibrium and decompose the change into income and substitution effects. Explain anything unusual in your diagram.

(f) Calculate the compensating and equivalent variations for the price increase in part (e). Explain any differences between this answer and your result for part (c).

Problem Set 5

(Due 10/8)

1. Compensating Variation

This exercise will show you how the methods you've learned so far can be used to calculate a compensating variation in a more general problem than that treated in the last problem set. Let's start by assuming a consumer buys two goods, x and y, according to preferences which can be represented by the following utility function: $U = xy$. Let the price of x be P_x and the price of y be P_y. The consumer's total income is I.

(a) How is the marginal rate of substitution related to the utility function? Prove that this consumer's marginal rate of substitution is $-y/x$. Be sure to show all your work.

(b) When a consumer is not at a corner, in general her equilibrium will have two important properties. What are these properties? Explain in nontechnical language how you know each will be true.

(c) Now show that the consumer's demands for x and y will be given by the equations $x = I/(2P_x)$ and $y = I/(2P_y)$.

(d) Derive an equation for the consumer's income elasticity of demand for x. Be sure to show all your work−just writing down the answer will receive no credit. Is x a normal or inferior good? How do you know?

(e) Graph the equilibrium for the case when I is \$12, P_x is \$1 and P_y is \$2. Show consumer's budget constraint, her indifference curve and the quantity of each good purchased. Label the intercepts of the budget constraint and label the indifference curve by calculating the consumer's utility.

(f) Notice that the consumer's optimal consumption of x and y derived in part (c) depends only on prices and income. Use this information and the consumer's utility function to show that the income needed to achieve a particular level of utility is given by $I = (4 P_x P_y U)^{1/2}$. This expression is very useful and is called the consumer's "expenditure function".

(g) Use the expenditure function to calculate the compensating variation when the price of x increases to \$1.44.

(h) Now use the information in part (g) to draw an accurate graph showing the consumer's initial equilibrium, final equilibrium, compensating variation and the income and substitution effects of this price change. How much does consumption of x change due to substitution? How much does it change due to the loss of purchasing power?

Problem Set 6

(Due 10/15)

1. Consumer Surplus

Consumer surplus can be a very handy way of measuring the effect of a price change on a consumer's welfare. However, you should always remember that it is only an approximation to more careful measures such as compensating or equivalent variation. This exercise will show you how consumer surplus can be used and how it can go wrong.

(a) Explain what consumer surplus is in nontechnical terms.

(b) Now explain the technical reason it differs from compensating or equivalent variation.

(c) Suppose Vic Video only consumes two things: Coke and arcade video games. Each time he plays a video game, he drinks exactly one Coke. Let Vic's income be I, the number of games he plays be x and the number of Cokes he drinks be y. If the price of video games is P_x and the price of Cokes is P_y, show that Vic's demand for video games is given by:

$$x = \frac{I}{P_x + P_y}$$

(d) Graph this demand curve accurately for the case when the price of video games is \$1, the price of a Coke is \$1 and Vic spends \$20 in total.

(e) For the numbers in part (d), Vic's compensated demand curve is $x^{comp} = 10$. Explain why this is so using a graph. If you can, use the expenditure function introduced in problem set 5 to confirm your graphical result.

(f) Now draw a diagram showing both Vic's ordinary and compensated demand curves. Show the change in Vic's consumer surplus that would occur if the price of video games rose to \$2. Also show the compensating variation and explain any differences between it and the consumer surplus.

Problem 2 is on the back.

2. Wage Subsidies

One of the proposals for helping what was once East Germany catch up to western standards of living is a wage subsidy. The idea is that the government would add a certain amount, say 1 Deutschmark (DM), to the hourly wage paid by private companies operating in selected industries. This question explores the implications of such a proposal.

(a) Suppose a worker in eastern Germany has 12 hours a day to divide between work and leisure. For each hour she works she gets paid 5 DM. Assuming she consumes leisure and an aggregate of all other goods, and that her indifference curves have the usual shape, show what her equilibrium might look like.

(b) Now copy your graph from part (a) and show what would happen if the government added 1 DM per hour to her wage.

(c) Using the diagram in part (b), show how much money the government will have to pay out in revenue after the policy takes effect. Compare that to the equivalent variation of the policy. (Hint: follow the approach we used in class to compare excise and lump sum taxes.)

(d) Is the wage subsidy a cost-effective way to increase the standard of living of this worker? Would your answer change if her labor supply curve were backward bending? Is there any reason why this policy might be preferred to an outright grant?

Problem Set 7

(Due 10/22)

1. Intertemporal Choice

Hal Headbanger is a heavy metal musician who must allocate his consumption between this year and next year. (Heavy metal being what it is, Hal doesn't expect to be around to worry about the year after that.) Hal's utility function is $U = x^{1/2}y^{1/2}$, where x is his consumption this year and y is his consumption next year.

(a) Suppose Hal expects to earn \$1 million this year and \$3 million next year and the interest rate is 10%. Find Hal's equilibrium consumption in each year and show the results in a graph. Does he borrow or lend this year? If so, how much?

(b) Now suppose that heavy metal musicians are known to be such bad credit risks that no one will lend to them. Show how this would change Hal's budget constraint and equilibrium.

(c) How could you find out the maximum interest rate Hal would pay to borrow money this year? What is that rate?

2. Present Value

Suppose the new state lottery passes and the first winner is Joe Bob Briggs, whose prize is a million dollars.

(a) Suppose the million dollars is to be paid out in 10 installments of \$100,000 each. If the interest rate is 5%, what is the present value of the prize?

(b) Now suppose the lottery commission offers Joe Bob a choice: the 10 payments in part (a) or \$50,000 per year forever. Joe Bob knows more about movies than finance and is nervous about the decision. What would you recommend he do? Why?

(c) What if the stream of payments in part (b) only lasted 30 years instead of forever. What would be your recommendation then? Hint: there is an easier way to solve this problem than adding up the present values of 30 payments one at a time.

3. Human Capital

Connie Careful is just about to graduate from college and is thinking of going on to law school. She is only interested in making the present value of her income stream as high as possible, and she can borrow and lend at an interest rate of 5%.

(a) If she doesn't go to law school she can earn \$30,000 a year for the next 45 years. What is the present value of this stream of income? The hint from problem 2, part c applies here as well.

(b) If she does go to law school, she has to spend 3 years studying full time (that is, not working) and paying tuition of \$20,000 a year. When she gets out, however, she'll be able to earn \$40,000 a year for 42 years. What is the present value of this part of the decision?

(c) What decision should Connie make? Would your advice be the same if the interest rate were a lot higher?

Problem Set 8

(Due 10/29)

1. Human Capital

Ned Nightschool cares only about his consumption this year and next year. His utility function is $U = x^{1/2}y^{1/2}$, where x is this year's consumption and y is next year's. At his current job, Ned can earn \$25,000 in each year. However, this year he also has the option of attending night school. Each class he takes costs \$2357, but it increases his income next year. If he takes N classes, the total increase in his income will be $\$7000N^{1/2}$.

(a) Graph Ned's budget constraint showing his feasible combinations of consumption in the two periods when he is not allowed to borrow or lend.

(b) If Ned attends N classes, his consumption this year will be given by $x = 25000 - 2357\,N$ and his consumption next year will be $y = 25000 + 7000\,N^{1/2}$. Use this information to show that Ned's budget constraint is given by: $y = 25000 + 7000((25000 - x)/2357)^{1/2}$.

(c) Now suppose that Ned can borrow or lend at an interest rate of 5%. Find his equilibrium number of classes and graph his new budget constraint.

(d) Finally, solve for Ned's equilibrium consumption in each of the two periods. How much money does he invest in human capital formation? How does his consumption change in each period relative to his endowment point?

2. Uncertainty

Former baseball great Pete Geranium is considering a bet on the outcome of the World Series between the Twins and the Braves. His local bookie, Jimmy, has offered him the following deal: if Pete bets \$1 on the Twins and the Twins win, Jimmy will return Pete's bet and pay him an additional \$2. However, if the Braves win, Pete loses his bet. Before betting, Pete's initial wealth is \$100.

(a) Graph Pete's budget constraint showing his consumption possibilities in the two states of the world: Twins win (T), and Braves win (B). Put his consumption in state B on the horizontal axis.

(b) If Pete bets \$W, his consumption in state B will be $C_B = 100 - W$ and in state T it will be $C_T = 100 + 2W$. Use this information to show that Pete's budget constraint can be written: $300 = 2C_B + C_T$

(c) Suppose Pete has inside information and knows that the probability of the Twins winning is 40%. What is the expected value of a \$1 bet? Is the bet actuarially fair? How much would Pete bet if he were risk-neutral?

(d) Now suppose Pete has a von Neumann-Morgenstern utility function of the form $U = \pi_B C_B^{1/2} + \pi_T C_T^{1/2}$, where π_T is the probability of a win by the Twins and π_B is the probability of a win by the Braves. If π_T is 0.4 (a 40% chance), how much will Pete bet?

(e) Graph the equilibrium from part (d). Is Pete risk-averse, risk-neutral, or risk-loving? How can you tell?