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Answers to Math Exercise

The answers to the math exercise are shown below. Each answer shows all steps leading to the solution, which is marked with "(*)".

Basic Algebra

Solve the following expression for y as a function of x:

$$y = 2y - 3x^2$$

Answer:

$$-y = -3x^2$$

$$y = 3x^2 \quad (*)$$

Systems of Equations

Solve the following two equations for the values of x and y:

$$2x + 3y = 4$$

$$x + 2y = 5$$

Answer:

$$x = 5 - 2y$$

$$2(5 - 2y) + 3y = 4$$

$$10 - 4y + 3y = 4$$

$$6 = y$$

$$x = 5 - 2(6) = -7$$

$$x = -7, y = 6 \quad (*)$$

Differentiation

Differentiate the following function with respect to x:

$$y(x) = ax^3 + \frac{b}{x}$$

Answer:

$$\frac{dy}{dx} = 3ax^2 - \frac{b}{x^2} \text{ (*)}$$

Maximization

Using calculus, find the value of x that maximizes the following function:

$$U(x) = 10 - (x-5)^2$$

Answer:

$$\frac{dU}{dx} = 0.$$

$$\frac{dU}{dx} = -2(x-5)$$

$$0 = -2x + 10$$

$$x = 5 \text{ (*)}$$

Partial Differentiation

Solve for the partial derivatives of the following function with respect to x and y:

$$U(x,y) = Ax^{\alpha}y^{\beta}$$

Answer:

$$\frac{\partial U}{\partial x} = \alpha Ax^{(\alpha-1)}y^{\beta} \text{ (*)}$$

$$\frac{\partial U}{\partial y} = \beta Ax^{\alpha}y^{(\beta-1)} \text{ (*)}$$

Total Differentiation

Write down the total differential of the following function:

$$U(x,y) = \frac{1}{3}(xy)^3$$

Answer:

$$dU = (xy)^2(ydx + xdy)$$

$$dU = x^2y^3dx + x^3y^2dy \quad (*)$$

Graphing

Using at least four points per curve, plot the following function for U=4 and U=9 (two curves) in the quadrant where x and y are both positive:

$$U(x,y) = xy^2$$

Answer:

See Figure 1

Inequalities

Sketch the region of the positive quadrant containing points satisfying the following inequality:

$$2x + y \leq 10$$

Answer:

See Figure 2

Figure 1

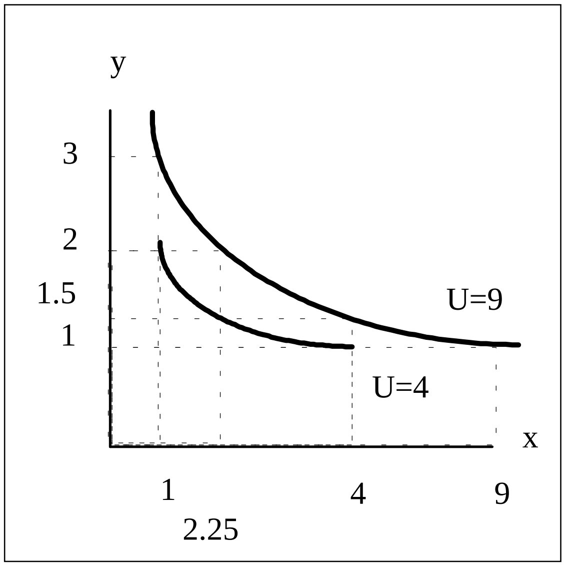
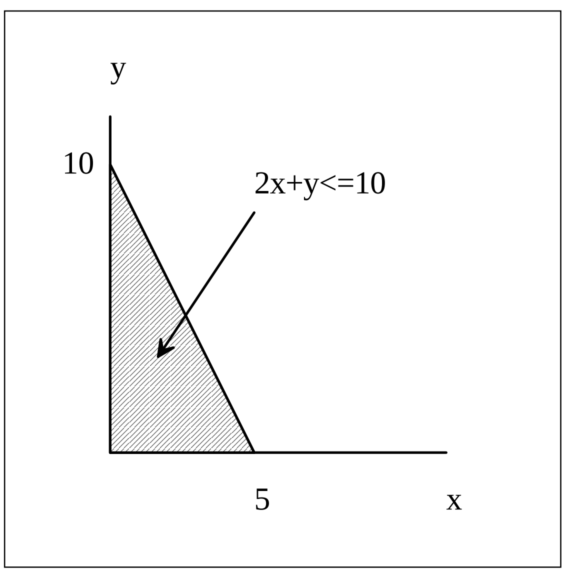


Figure 2



Constrained Maximization (OPTIONAL)

Find the values of x and y that maximize the function:

$$f(x,y) = xy$$

subject to the constraint:

$$x + y = m$$

where m is a constant.

Answer:

$$y = m - x$$

$$f = x(m-x) = mx - x^2$$

$$\frac{df}{dx} = m - 2x$$

$$0 = m - 2x$$

$$x = \frac{m}{2}$$

$$y = m - \frac{m}{2} = \frac{m}{2}$$

$$x = \frac{m}{2}, \quad y = \frac{m}{2} \ (*)$$

Integration (OPTIONAL)

Integrate the following expression with respect to t, where A and r are constants:

$$\int_0^\infty Ae^{-rt}dt$$

Answer:

$$= \frac{1}{-r}Ae^{-rt} \,\Big|_0^\infty$$

$$= \lim_{t \to \infty} \frac{1}{-r}Ae^{-rt} - \frac{1}{-r}Ae^{0}$$

$$= \frac{A}{r} \ (*)$$