Exercise AU-501

Imperfect information and health policy

The Economic Skills Project

1 Problem

Problem

A foundation is considering two policies, C and N, to address a public health problem. C is a conventional approach that would cost $20 million and produce $50 million in benefits. N is a new approach that would cost $10 million but has uncertain benefits: theres a 20% chance benefits would $110 million (high or H) and an 80% chance they would be $20 million (low or L).

In addition, the foundation could fund a pilot study to see whether H or L is true before the main project is carried out. However, the pilot might be wrong: there is a 20% chance it would report that benefits would be L when theyd actually be H, and a 60% chance it would report benefits would be H when theyd actually be L.

Problem, continued

What is the maximum amount the foundation would be willing to pay the pilot study, if anything? If it would be better not to carry out the pilot indicate which policy the foundation would adopt.

You may assume the foundation is risk neutral and that everything happens immediately so no present value calculations are needed.

2 Answer

Answer

The foundation would be willing to pay up to $1.6M for the pilot study. If the cost were higher than that, its next best alternative would be policy C.

3 Method

Solution method

Here's one approach:

1. Draw the decision tree out to the information sets from the pilot study.
2. Calculate the conditional probabilities of H and L given the pilot's results.
3. Redraw the tree to show the probability of each type of report.
4. Evaluate the tree from right to left.

4 Solution

4.1 Step 1

Draw the decision tree out to the information sets

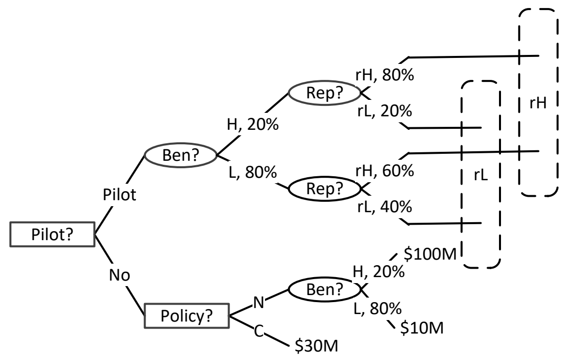
The pilot study will return one of two possible reports:

1. That benefits will be H: call that "rH" for "reports H".
2. That benefits will be L: call that "rL" for "reports L".

Since the test is imperfect there will be multiple ways that each report could arise. Because the foundation sees only the report, and not the events that led to it, it can't tell the difference between nodes that have the same report. To show that, they are grouped together by dotted lines into "information sets" in the diagram.

The tree out to the rH and rL information sets is shown on the next page.

Draw the decision tree out to the information sets



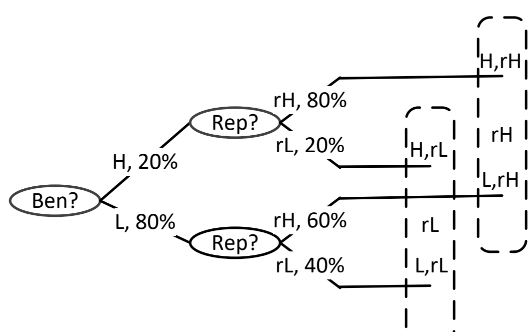
4.2 Step 2

Calculate the conditional probabilities

The next step is to calculate the overall probability of each kind of report, rH and rL, and then to calculate the probability of H or L being true conditional on having received either rH or rL.

To do that it's convenient to zoom in on the part of the tree associated with the pilot study and assign names to the endpoints. The endpoint where benefits are H and the report is rH will be called "H,rH" and the other nodes will be named similarly. The result is shown on the next page.

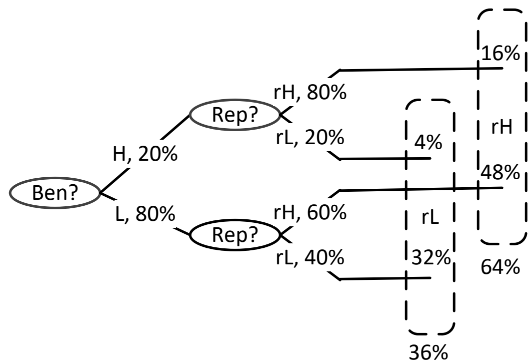
Calculate the conditional probabilities



The probability of ending up at any one of the endpoints is the product of the probabilities along the way.

For example, the probability of arriving at H,rH is $0.2 \times 0.8 = 0.16$ or 16%.

Calculate the conditional probabilities



Here's the result of filling in the four endpoint probabilities.

The overall probabilities of ending up with an rH or an rL report are shown below each dotted region.

Calculate the conditional probabilities

It's now possible to calculate the conditional probabilities of H and L given the report received. The probability that benefits are actually high (H) given a report that they were high in the trial (rH) is:

$$Pr(H|rH) = \frac{Pr(H, rH)}{Pr(rH)} = \frac{0.16}{0.64} = 0.25$$

The probability that benefits are actually low (L) despite an rH report from the trial is:

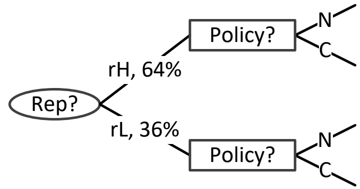
$$Pr(L|rH) = \frac{Pr(L, rH)}{Pr(rH)} = \frac{0.48}{0.64} = 0.75$$

The probabilities of H and L given an rL report are calculated in a similar way.

4.3 Step 3

Redraw the tree showing probabilities of reports

The analysis in step 2 allows the pilot branch to be simplified to a chance node with two possible results: rH and rL. Each of these branches can now be joined to a choice between the policies.



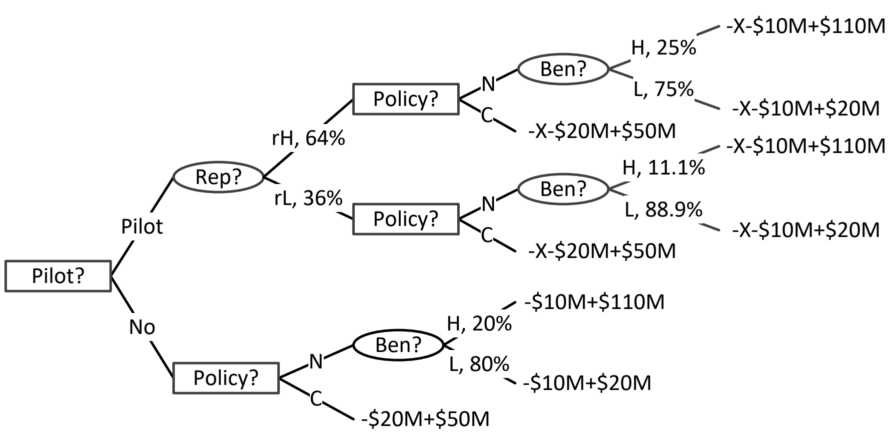
Redraw the tree showing probabilities of reports

The next step is to redraw the pilot study branch of the tree by doing the following:

- Put in the revised trial node and policy choices above;
- Add a chance node for H and L after each N branch.
- Use the conditional probabilities for the new chances of H or L.
- Redraw the whole tree with all payoffs;

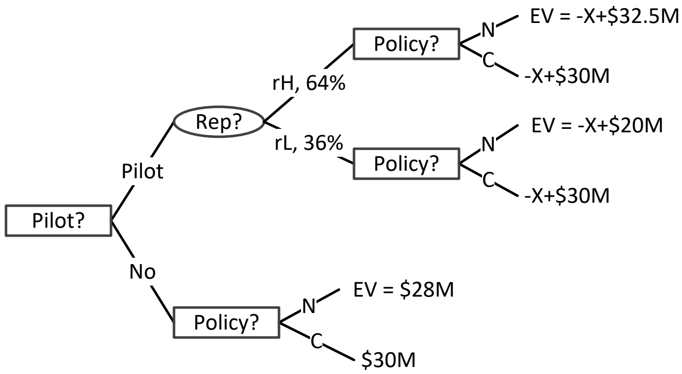
The new diagram is shown on the next page with amount spent on the trial is shown as X.

Redraw the tree showing probabilities of reports



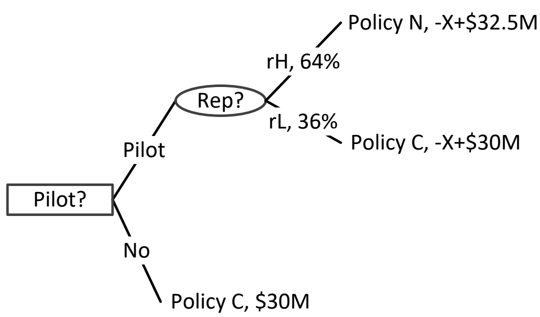
Evaluate the tree from right to left

The three right-most nodes are all chance nodes so they are evaluated by computing their EVs. The result is:



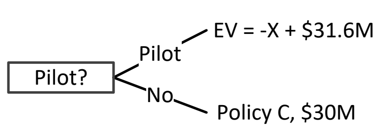
Evaluate the tree from right to left, continued

The new right-most nodes are all choices so they are evaluated by picking the action with the best payoff. The result:



Evaluate the tree from right to left, continued

Evaluating the report node by computing its expected value gives:



Evaluate the tree from right to left, continued

The pilot project will be the best option when its payoff is at least as high as the best payoff without it. Solving for the value of X that makes the payoffs equal:

$$-X + \$31.6M = \$30M$$

Thus, the foundation's maximum willingness to pay for the trial will be:

$$X = \$1.6M$$

Done!

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