

# Exercise CI-102

Borrowing by an individual with Cobb-Douglas preferences

## The Economic Skills Project

### 1 Problem

#### Problem

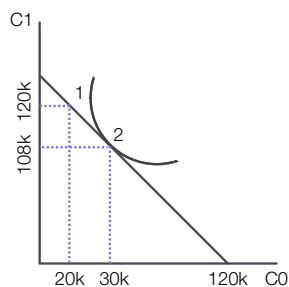
An individual is concerned about his consumption in two periods, 0 and 1. In period 0 his income will be \$20,000 and in period 1 it will be \$120,000. His utility function and demand equations for consumption are given below, where PVI is the present value of his income stream. He can borrow or save at an interest rate of 20%. How much does he borrow or save in 0? Draw a diagram illustrating his equilibrium.

$$U = C_0^{0.25} C_1^{0.75}$$
$$C_0 = \frac{0.25 \cdot PVI}{P_0}, \quad C_1 = \frac{0.75 \cdot PVI}{P_1}$$

### 2 Answer

#### Answer

He borrows \$10,000 and moves from his endowment at point 1 to his preferred consumption bundle at point 2.



### 3 Method

#### Solution method

Here's one approach:

1. Calculate the present value of income.
2. Use the demand equations to compute  $C_0$  and  $C_1$ .
3. Compare  $C_0$  to  $I_0$  to determine his borrowing.
4. Draw the diagram.
5. Check the result.

### 4 Solution

#### 4.1 Step 1

##### Calculate the present value of income

The present value of the income stream is:

$$PVI = I_0 + \frac{I_1}{1+r}$$

Inserting  $r$ ,  $I_0$ , and  $I_1$ :

$$PVI = \$20,000 + \frac{\$120,000}{1.2}$$

Calculating it:

$$PVI = \$120,000$$

#### 4.2 Step 2

##### Use the demand equations

The prices,  $P_0$  and  $P_1$ , of consumption in the two periods are the coefficients on  $C_0$  and  $C_1$  in the intertemporal budget constraint:

$$C_0 + \frac{C_1}{1+r} = PVI$$

Thus,  $P_0 = 1$  and  $P_1 = \frac{1}{1+r}$ . Inserting those and the value of PVI into the demands:

$$C_0 = \frac{0.25 \cdot \$120,000}{1} = \$30,000$$

$$C_1 = \frac{0.75 \cdot \$120,000}{\left(\frac{1}{1.2}\right)} = \$108,000$$

### 4.3 Step 3

#### Compare $C_0$ to $I_0$ to determine borrowing

Since his consumption in 0 is higher than his income, he'll borrow in period 0. The amount will be:

$$B = C_0 - I_0$$

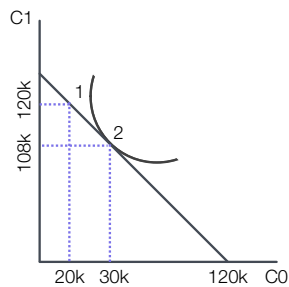
Filling in the numbers:

$$B = \$30,000 - \$20,000 = \$10,000$$

### 4.4 Step 4

#### Draw the diagram

His endowment is point 1:  $C_0 = \$20k$  and  $C_1 = \$120k$ . His preferred consumption bundle is point 2:  $C_0 = \$30k$  and  $C_1 = \$108k$ .



### 4.5 Step 5

#### Check the result

One way to check the result is to calculate the present value of his consumption bundle:

$$C_0 + \frac{C_1}{1.1} = \$30,000 + \frac{\$108,000}{1.2} = \$120,000$$

Since that equals the present value of his income, it passes the check.

An alternative check is to compute his disposable future income by deducting what he has to repay to make sure it matches his consumption. His loan repayments will be  $\$10,000 \cdot 1.2 = \$12,000$ . Deducting that from  $I_1$  gives  $\$108,000$ . Since it exactly matches  $C_1$ , the result passes the second test as well.

Everything checks - done!