Exercise CN-205

Choosing whether to buy information

The Economic Skills Project

1 Problem

Problem

Alice is considering buying a used car. It would cost $10,000 but its quality is unknown. There's a 25% chance it's good (G) and worth $15,000, but a 75% chance it's bad (B) and worth only $5,000.

Alice could also have the car tested for $400 before she decides whether to buy it. The test would determine the quality of the car with certainty.

If Alice is risk-neutral and cares only about expected value, what will she choose, and what will her expected net payoff be?

2 Answer

Answer

She'll buy the test and then buy the car if the test says it's good. Her expected net payoff is $850.

3 Method

Solution method

Here's one approach:

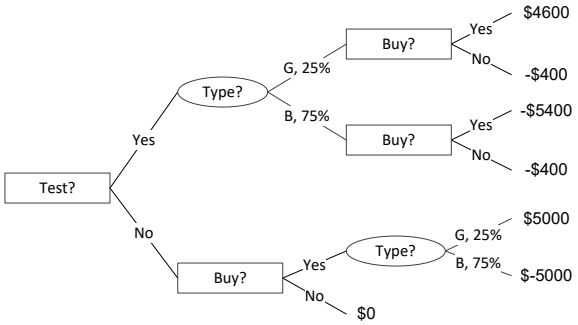
1. Draw the decision tree.
2. Repeatedly evaluate the nodes from right to left.

4 Solution

4.1 Step 1

Draw the decision tree

Here's her decision tree; the payoffs are explained on the next page.



Decision tree payoffs

All of the payoffs in the tree are Alice's net outcomes at the end points. For example, the top endpoint occurs when she buys the test for $400, finds out the car is good, buys the car for $10,000, and ends up with a $15,000 car. That gives her a net payoff of:

$$-\$400 - \$10,000 + \$15,000 = \$4,600$$

Similarly, if she decides not to buy the test, buys the car anyway, and ends up with a good car, she'll be at the fifth endpoint from the top and her payoff will be:

$$-\$10,000 + \$15,000 = \$5,000$$

If she doesn't buy the test or buy the car, her net payoff is zero because her situation is unchanged by the decision.

4.2 Step 2

Evaluating the nodes from right to left

There are three nodes at the far right side of the diagram: the two choice nodes on the branch where she chooses the test, and the chance node where she buys the car without the test.

The two choice nodes are evaluated by picking the action with the best payoff. For the top node, where the car is good, she'd buy it and have a payoff of $4,600. For the next node, where the car is bad, she'd walk away from the deal with a payoff of -$400.

Finally, the chance node at the bottom is evaluated by computing its expected value:

$$0.25(\$5,000) + 0.75(-\$5,000) = -\$2,500$$

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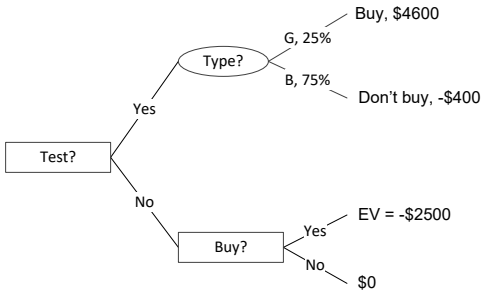
Evaluating the nodes from right to left, continued

After the first group of nodes is evaluated the tree can be simplified to:



Evaluating the nodes from right to left, continued

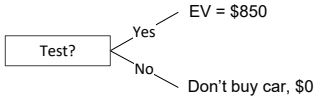
There are now two nodes at the far right. The top is a chance node with an expected value of:

$$0.25(\$4,600) + 0.75(-\$400) = \$850$$

The lower node is a choice and the highest payoff is not to buy the car, with a value of $0.

Evaluating the nodes from right to left, continued

Simplifying the tree one more time gives:



Evaluating that node gives the final outcome: the best option is to buy the test, and it has an expected value of $850.

Done!