

Exercise CW-152

Deriving a Stone-Geary expenditure function

The Economic Skills Project

1 Problem

Problem

A household consumes two goods, X and Y, and has preferences that can be represented by the Stone-Geary utility function shown below. The corresponding demand equations are also shown.

$$U = (X - 20)^{0.5} \cdot (Y + 20)^{0.5}$$
$$X = 20 + \frac{0.5M - 10P_X + 10P_Y}{P_X}, \quad Y = -20 + \frac{0.5M - 10P_X + 10P_Y}{P_Y}$$

Derive the expenditure function for the household.

2 Answer

Answer

Here's one way to write the function:

$$M = 2U \cdot P_X^{0.5} P_Y^{0.5} + 20P_X - 20P_Y$$

3 Method

Solution method

Here's one approach:

1. Substitute the demand equations into the utility function.
2. Collect terms in M.
3. Solve the resulting equation for M.

4 Solution

4.1 Step 1

Substitute the demand equations into the utility function

The utility function involves $X - 20$ and $Y + 20$ so it's helpful to compute them first:

$$X - 20 = 20 + \frac{0.5M - 10P_X + 10P_Y}{P_X} - 20 = \frac{0.5M - 10P_X + 10P_Y}{P_X}$$
$$Y + 20 = -20 + \frac{0.5M - 10P_X + 10P_Y}{P_Y} + 20 = \frac{0.5M - 10P_X + 10P_Y}{P_Y}$$

Inserting them into the utility function:

$$u = \left(\frac{0.5M - 10P_X + 10P_Y}{P_X} \right)^{0.5} \left(\frac{0.5M - 10P_X + 10P_Y}{P_Y} \right)^{0.5}$$

4.2 Step 2

Collect terms in M

The numerators of are identical square roots so the function can be simplified to:

$$u = \frac{0.5M - 10P_X + 10P_Y}{P_X^{0.5} P_Y^{0.5}}$$

4.3 Step 3

Solve for M

Multiplying both sides by $P_X^{0.5} P_Y^{0.5}$:

$$u \cdot P_X^{0.5} P_Y^{0.5} = 0.5M - 10P_X + 10P_Y$$

Moving the remaining price terms to the left side:

$$u \cdot P_X^{0.5} P_Y^{0.5} + 10P_X - 10P_Y = 0.5M$$

Multiplying by 2 to solve for M:

$$M = 2u \cdot P_X^{0.5} P_Y^{0.5} + 20P_X - 20P_Y$$

Done!