Exercise CW-252

Income and substitution effects for Stone-Geary preferences

The Economic Skills Project

1 Problem

Problem

A household has the Stone-Geary utility function, demand curves, and expenditure function shown below:

$$U = (X - 200)^{0.5} (Y + 100)^{0.5}$$
$$M = 2U \cdot (P_X)^{0.5} (P_Y)^{0.5} + 200P_X - 100P_Y$$
$$X = 200 + \frac{0.5 (M - 200P_X + 100P_Y)}{P_X}$$
$$Y = -100 + \frac{0.5 (M - 200P_X + 100P_Y)}{P_Y}$$

Problem, continued

Initially, $P_X = \$20$, $P_Y = \$10$, and $M = \$20,000$. The government is considering a policy that would raise the price P_X to \$25. There would be no change in M or P_Y. What are the income and substitution effects of the policy?

2 Answer

Answer

Here's the solution:

- Substitution effect: -45 units of X
- Income effect: -60 units of X

3 Method

Solution method

Here's one approach:

1. Use the demand equations to compute X_1 and Y_1.
2. Use the utility function to compute U_1.
3. Use the demand equation to compute X_2.
4. Use the expenditure function to compute M_3.
5. Use M_3 to compute X_3.
6. Compare X_3 to X_1 and X_2 to compute the effects.
7. Check the result.

4 Solution

4.1 Step 1

Use the demand equations to compute X_1 and Y_1

Inserting the initial values of M_1, P_{X1}, and P_{Y1} into the demands gives:

$$X_1 = 200 + \frac{0.5\,(\$20k - 200 \cdot \$20 + 100 \cdot \$10)}{\$20} = 625$$

$$Y_1 = -100 + \frac{0.5\,(\$20k - 200 \cdot \$20 + 100 \cdot \$10)}{\$20} = 750$$

4.2 Step 2

Use the utility function to compute U_1

Using X_1 and Y_1 to compute U_1:

$$U = (625 - 200)^{0.5}\,(850 + 100)^{0.5} = 601.04$$

4.3 Step 3

Use the demand equation to compute X_2

Inserting the new values of M_2 (unchanged), P_{X2}, and P_{Y2} (unchanged) into the demands gives:

$$X_2 = 200 + \frac{0.5\,(\$20k - 200 \cdot \$25 + 100 \cdot \$10)}{\$25} = 520$$

4.4 Step 4

Use the expenditure function to compute M_3

Inserting U_1 and P_{X2} and P_{Y2} into the expenditure function gives M_3, the expenditure needed to get the original utility at the new prices:

$$M_3 = 2U_1 \cdot (P_{X2})^{0.5}\,(P_{Y2})^{0.5} + 200P_{X2} - 100P_{Y2}$$

$$M_3 = 2\,(601.04) \cdot (\$25)^{0.5}\,(\$10)^{0.5} + 200 \cdot \$25 - 100 \cdot \$10$$

$$M_3 = \$23,007$$

4.5 Step 5

Use M_3 **to compute** X_3

Inserting M_3 and P_{X2} into the demand equation gives X_3, the amount the household would consume if it had been compensated and remained on the original indifference curve:

$$X_3 = 200 + \frac{0.5\,(\$23,007 - 200 \cdot \$25 + 100 \cdot \$10)}{\$25} = 580$$

4.6 Step 6

Compare X_3 **to** X_1 **and** X_2 **to compute the effects**

The substitution and income effects, ΔX_S and ΔX_I, are defined as follows:

$$\Delta X_S = X_3 - X_1$$

$$\Delta X_I = X_2 - X_3$$

Inserting the values of X_1, X_2, and X_3:

$$\Delta X_S = 580 - 625 = -45$$

$$\Delta X_I = 520 - 580 = -60$$

4.7 Step 7

Check the result

The sum of the income and substitution effects, $\Delta X_S + \Delta X_I$, should be equal to the actual change in X. The actual change is:

$$X_2 - X_1 = 520 - 625 = -105$$

The sum of the income and substitution effects is:

$$\Delta X_S + \Delta X_I = -45 - 60 = -105$$

Everything checks - done!

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