

Exam 1
Spring 2006
Version G Solution

Part 1: Single Source Pollution (20 points)

Suppose that electricity can be produced by nuclear power at a constant marginal cost of \$10 per unit (that is, $MC = \$10$). A city's demand for electricity from nuclear power is given by the equation: $P = \$15 - Q/1,000,000$.

- (1) Please determine the market equilibrium price and quantity of electricity.

$$P = 15 - Q/1,000,000$$

$$P = MC$$

$$P = 10$$

$$10 = 15 - Q/1,000,000$$

$$5 = Q/1,000,000$$

$$Q = 5,000,000$$

- (2) Suppose each unit of electricity generated produces 1 gram of radioactive waste. The waste remains safely inside the reactor for 20 years. After 20 years, the reactor core is moved to a storage site. From year 21 on (forever), there is a 1/100,000,000 chance (each year) that a person will come into contact with the waste. If that happens, the damages are \$6 million. Calculate the expected present value of the damages associated with one unit of electricity. Be sure to show all your work. You may assume the interest rate is 5%.

EV of damages from 21 on:

$$EV = (1/100 \text{ million}) * (\$6 \text{ million}) = \$0.06$$

PV of expected damages:

$$\text{As of year 20: } \$0.06/0.05 = \$1.20$$

$$\text{As of year 0: } \$1.20/(1.05)^{20} = \$0.45$$

- (3) Using your answer to question (2), find the efficient price and quantity of electricity. If we were to achieve this by taxing nuclear power, what should the tax rate be? What will be the total dollar effect of this on consumer surplus? On government revenue? On the externality problem? What is the overall welfare gain?

Efficient P and Q:

$$MSC = MC + MC_{\text{ext}}$$

$$MSC = \$10 + \$0.45 = \$10.45$$

$$P_{\text{eff}} = \text{MSC} = \$10.45$$

$$P_{\text{eff}} = 15 - Q_{\text{eff}}/1,000,000$$

$$\$10.45 = \$15 - Q_{\text{eff}}/1,000,000$$

$$Q_{\text{eff}} = 4,550,000$$

Tax:

$$\text{Tax} = \text{MC}_{\text{ext}} = \$0.45$$

Effect on CS:

$$\text{CS} = - (\$0.45 * 4,550,000 + (1/2)*(\$0.45)*(5,000,000-4,550,000))$$

$$\text{CS} = - (\$2.048 \text{ million} + \$0.101 \text{ million})$$

$$\text{CS} = - \$2.149 \text{ million}$$

Effect on revenue:

$$\text{Rev} = \$0.45 * 4,550,000 = \$2.048 \text{ million}$$

Effect on the externality:

$$\text{Reduced by: } \$0.45 * (5,000,000 - 4,550,000)$$

$$\text{Savings} = \$0.202 \text{ million}$$

Overall welfare gain:

$$\text{Change in CS:} \quad - \$2.149 \text{ million}$$

$$\text{Change in Revenue:} \quad + \$2.048 \text{ million}$$

$$\text{Change in Externality:} \quad + \$0.202 \text{ million}$$

$$\text{Net:} \quad + \$0.101 \text{ million}$$

Part 2: Multiple Source Pollution (30 points)

Suppose a city is concerned about a new air pollutant. The pollutant is currently uncontrolled and 200 tons are emitted each year. The emissions come from two sources, each of which is responsible for 100 units. Source 1's marginal abatement cost is given by $MC_1 = 4 * Q_1$, where Q_1 is the amount of abatement it does. Source 2's marginal abatement cost is $MC_2 = 8 * Q_2$. The marginal benefits of abatement are believed to be given by a function of the form: $MB = A - B * Q_a$, where A and B are parameters and Q_a is the total amount of abatement.

- (4) A study reports that the marginal benefit for an improvement in air quality from the uncontrolled level (i.e., when $Q_a = 0$) would be \$300. The study also reports that if the pollution level were reduced to 150 tons, the marginal benefit of abatement would fall to \$250. Determine the efficient level of abatement. How much should source 1 clean up? Source 2?

Finding the MBa curve

Basic information:

$$MB = A - B \cdot Q_a$$

$$\text{If } Q_a = 0, MB_a = \$300$$

$$\text{If } Q_a = 200 - 150 = 50, MB_a = \$250$$

$$\$300 = A - B \cdot 0$$

$$\$250 = \$300 - B \cdot 50$$

$$B = 50/50 = 1$$

$$MB = \$300 - Q_a$$

Finding the MCa curve

$$MC_1 = 4 \cdot Q_1$$

$$MC_2 = 8 \cdot Q_2$$

$$Q_1 = MC_1 / 4$$

$$Q_2 = MC_2 / 8$$

$$Q_a = Q_1 + Q_2$$

$$Q_a = (MC_1 / 4) + (MC_2 / 8)$$

$$Q_a = (2 \cdot MC_1 + MC_2) / 8$$

$$Q_a = (3 \cdot MC) / 8$$

$$MC = 8 \cdot Q_a / 3$$

Finding the efficient point

$$MB = \$300 - Q_a$$

$$MC = 8 \cdot Q_a / 3$$

$$\$300 - Q_a = 8 \cdot Q_a / 3$$

$$\$300 = 8 \cdot Q_a / 3 + Q_a = (11/3) \cdot Q_a$$

$$Q_a = 900/11 = 81.8$$

$$MB = \$300 - 81.8 = \$218.2$$

$$MC = 8 \cdot 81.8 / 3 = \$218.1 \text{ (off due to rounding)}$$

Abatement by sources:

$$Q_1 = \$218.2 / 4 = 54.55$$

$$Q_2 = \$218.2 / 8 = 27.28$$

- (5) Design a tradable permit policy that would achieve the efficient amount of abatement while spreading the overall cost equally between the two firms. How many permits would you distribute to each firm? What would the price of a permit be in equilibrium?

$$\text{Cost 1} = (1/2) * 54.55 * \$218.2 = \$5,951$$

$$\text{Cost 2} = (1/2) * 27.28 * \$218.2 = \$2,976$$

$$\text{Price of permit} = MC_a = \$218.2$$

Firm	Abate Cost	Equal Cost	Change	Sell Permits	Buy Permits	Permits Needed	Permits Granted
1	5,951	4,463.5	-1,487.5	6.82		45.45	52.27
2	2,976	4,463.5	+1,487.5		6.82	72.72	65.90
Total	8,927	8,927	0			118.17	118.17

Part 3: Pollution Control Under Uncertainty (30 points)

Suppose that a particular water pollutant causes \$50 of damage per ton. Two sources emit the pollutant and each is currently generating 100 tons (total emissions = 200 tons). Source 1 is known to be able to reduce its emissions at a marginal cost given by $MC_1 = 1 * Q_1$. Source 2's abatement costs are not certain. One possibility is that $MC_2 = 2 * Q_2$ but it's also possible that $MC_2 = 5 * Q_2$. It cannot be determined in advance which of the MC_2 curves is correct.

- (6) If it were certain that source 2 had the first marginal cost curve ($MC_2 = 2 * Q_2$), calculate each of the following: the efficient total amount of abatement; the amount of abatement that should be done by each source; the emissions tax that would get to efficiency; the quantity of permits that would achieve efficiency; and the market-clearing price of a permit if a permit policy were used.

$$MB_a = \$50$$

$$\text{Source 1: } MC_1 = 1 * Q_1 = MB_a$$

$$\text{Source 2: } MC_2 = 2 * Q_2 = MB_a$$

$$\text{Source 1: } Q_1 = 50 \text{ tons}$$

$$\text{Source 2: } Q_2 = 50/2 = 25 \text{ tons}$$

Total abatement: $Q_1 + Q_2 = 75$ tons

Emissions tax needed: $T = MB_a = \$50$

Q of permits = original emissions – abatement

Q of permits = 200 tons – 75 tons = 125 tons

Market clearing price of a permit: \$50 (MC_a at the efficient pattern of abatement)

In the remaining questions, suppose that one of the policies has been imposed and source 2 turns out to have the second marginal cost curve ($MC_2 = 5 * Q_2$).

- (7) Suppose the emissions tax was imposed. How much abatement will be done by each source? Is this efficient? Discuss.

Source 1: $MC_1 = \$1 * Q_1 = T = \50

Source 1: $Q_1 = 50$ tons

Source 2: $MC_2 = \$5 * Q_2 = T = \50

Source 2: $Q_2 = \$50 / \$5 = 10$ tons

Total abatement: $Q_1 + Q_2 = 60$ tons

This *is* efficient: $MC_1 = MC_2 = MB_a$. The marginal costs of abatement are equal across the sources, and each MC is equal to the MB of abatement. The $MB_a = MC_a$ condition shows that the right amount of abatement is being done, and the $MC_1 = MC_2$ condition shows that it is allocated efficiently across the sources.

- (8) Suppose the permit policy was imposed and each of the sources has been given half of the permits. How much abatement will be done by each source? Is this efficient? Discuss. Will there be any sales of permits from one source to another? If so, calculate the value of the permit sales (assuming that the permit market is perfectly competitive).

Each source gets $125/2 = 62.5$ permits

Total abatement = 75 tons

Finding the overall MC curve for abatement:

$Q_a = Q_1 + Q_2$

$Q_a = (MC_1/1) + (MC_2/5)$

$Q_a = (6/5) * MC$

$MC = Q_a * 5/6 = 75 * 5/6 = \62.50

Source 1: $Q_1 = 62.5$ tons

Source 2: $Q_2 = 62.5/5 = 12.5$ tons

Total: $Q_1 + Q_2 = 75$ tons

Not efficient: $MC_a > MB_a$ – too much abatement is being done. The efficient amount

would be 60 tons rather than 75 (from part 7).

Source 1 starts with 62.5 permits but needs only $100 - 62.5 = 37.5$; sells 25 permits

Source 2 starts with 62.5 permits but needs $100 - 12.5 = 87.5$; must buy 25 permits.

Price of a permit will be \$62.5 (equal to the MC)

Value of permit sales = $25 * \$62.5 = \$1,562.5$