

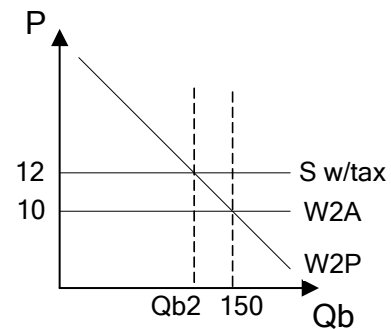
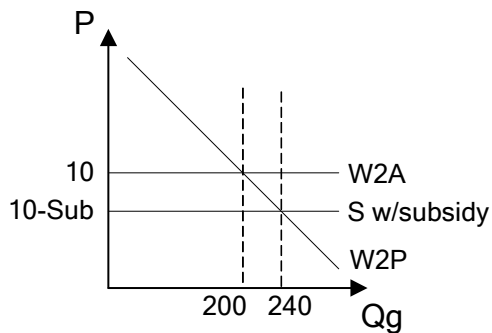
Solution to Exam 1
Spring 2007

Here are notes on the solution. Some of the graphs may be omitted and the explanations are a bit terse. If you have any questions, please don't hesitate to stop by during office hours or the lab session to talk over things in detail.

Part 1

1(a) Finding the new equilibria

The initial and target equilibria in each market are shown below (G on the left):



The first step is to calculate the subsidy needed in market G. Using the demand elasticity to find the price change needed to increase Q to the target level:

$$\eta = \frac{\% \Delta Q}{\% \Delta P}$$

$$\% \Delta P = \frac{\% \Delta Q}{\eta}$$

$$\% \Delta P = \frac{(240-200)/200}{(-2)}$$

$$\% \Delta P = \frac{20\%}{(-2)}$$

$$\% \Delta P = -10\%$$

$$\Delta P = -0.1 * \$10 = -\$1$$

$$P \text{ with subsidy} = \$10 - \$1 = \$9$$

The subsidy per unit of good G must be \$1 and the price must be \$9. The cost of the subsidy will be $\$1 * 240 = \240 .

The next step is to calculate the effect of the tax on Q_b . Using the elasticity:

$$\eta = \% \Delta Q / \% \Delta P$$

$$\% \Delta Q_b = \eta * \% \Delta P_b$$

$$\% \Delta Q_b = (-1) * ((12-10)/10) = -1 * 20\% = -20\%$$

$$\Delta Q_b = -0.2 * 150 = -30$$

$$Q_{b2} = 150 - 30 = 120$$

Revenue from the tax will be $\$2 * 120 = \240 . Since the expenditure on the subsidy exactly matches the revenue raised by the tax, the cross-subsidy will work without creating a budget surplus or deficit.

1(b) Changes in surplus

The change in CS in each market can be calculated as follows:

$$\Delta CS_g = \$1 * 200 + 0.5 * \$1 * 40 = \$220$$

$$\Delta CS_b = -(\$2 * 120 + 0.5 * \$2 * 30) = -\$270$$

Deadweight loss is just the sum (difference between lost CS in market B and gained CS in market A):

$$DWL = \$220 - \$270 = -\$50$$

Checking by calculating the DWL triangles in the two markets separately:

$$DWL_g = 0.5 * \$1 * 40 = \$20$$

$$DWL_b = 0.5 * \$2 * 30 = \$30$$

$$DWL = DWL_g + DWL_b$$

Part 2

2(a) Market equilibrium

The first step is to rearrange each W2P equation to find the quantity demanded by a given individual of each type:

$$W2Pa = 500 - Qa$$

$$W2Pa = P = 500 - Qa$$

$$Qa = 500 - P$$

$$W2Pb = 500 - 5*Qb$$

$$W2Pb = P = 500 - 5*Qb$$

$$Qb = (500 - P)/5$$

The market demand is the sum of the individual demands. Since there are 10 type-A buyers and 20 type-B buyers, the total Qd demanded will be:

$$Qd = 10*Qa + 20*Qb$$

$$Qd = 10*(500 - P) + 20*(500 - P)/5$$

$$Qd = 5000 - 10*P + 2000 - 4*P$$

$$Qd = 7000 - 14*P$$

The supply curve is:

$$W2A = P = Qs/6$$

$$Qs = 6*P$$

Finding the equilibrium:

$$Qd = Qs$$

$$7000 - 14*P = 6*P$$

$$7000 = 20*P$$

$$P = 350$$

$$Qd = 7000 - 14*350 = 7000 - 4900 = 2100$$

$$\text{Checking: } Qs = 6*350 = 2100$$

The question does not ask for the individual Q's but they are straightforward to calculate and are a useful check:

$$Q_a = 500 - 350 = 150$$

$$Q_b = (500 - 350)/5 = 30$$

$$Q_d = 10*150 + 20*30 = 1500 + 600 = 2100$$

2(b) Effect of a \$200 tax

With the tax, the seller will only supply the good when the buyers pay $P = W2A + \$200$. Using that to find the supply curve with the tax:

$$P = W2A + \$200$$

$$P = Q_s/6 + \$200$$

$$Q_s = 6*P - \$1200$$

Finding the equilibrium:

$$Q_d = Q_s$$

$$7000 - 14*P = 6*P - 1200$$

$$8200 = 20*P$$

$$P = \$410$$

$$Q_d = 7000 - 14*410 = 1260$$

$$\text{Check: } Q_s = 6*410 - \$1200 = 1260$$

Part 3

Policy 1 would raise the price of good X by \$20 and have no effect on good Y. Using the elasticity to find the change in the quantity of good X:

$$\eta = \% \Delta Q / \% \Delta P$$

$$\% \Delta Q_x = \eta * \% \Delta P_x$$

$$\% \Delta Q_x = (-1) * ((120 - 100) / 100) = (-1) * (20\%) = -20\%$$

$$\Delta Q_x = -0.2 * 1000 = -200$$

$$Q_x = 1000 - 200 = 800$$

Revenue raised: $\$20 * 800 = \$16,000$. Deadweight loss: $0.5 * \$20 * (1,000 - 800) = \$2,000$.

Policy 2 would raise the price of each good by \$10. Since each good originally sells for \$100, that's an increase of 10%. Using the elasticity formula to find the changes in Q_x and Q_y (both will be the same):

$$\% \Delta Q_x = \eta * \% \Delta P_x = (-1) * (10\%) = -10\%$$

$$\% \Delta Q_y = \eta * \% \Delta P_y = (-1) * (10\%) = -10\%$$

$$\Delta Q_x = -0.1 * 1000 = -100$$

$$\Delta Q_y = -0.1 * 1000 = -100$$

$$Q_x = 1000 - 100 = 900$$

$$Q_y = 1000 - 100 = 900$$

Revenue raised: $\$10 * 900 + \$10 * 900 = \$18,000$. Deadweight loss: $0.5 * \$10 * (1,000 - 900) + 0.5 * \$10 * (1,000 - 900) = \$500 + \$500 = \$1,000$.

Policy 2 is unambiguously better: it raises \$2,000 more revenue and has \$1,000 less deadweight loss. That turns out to be true in general, by the way: several small taxes are almost always better than one big one.