Peter J. Wilcoxen
PPA 723, Managerial Economics
Department of Public Administration
The Maxwell School, Syracuse University

Solution to Exam 2
Fall 2006

Here are notes on the solution. The graphs are omitted and the explanations are very terse. If you have any questions, please don't hesitate to stop by during office hours or the lab session to talk over things in detail.

Question 1a

Intercepts of the budget constraint:

X axis = $800/$2 = 400

Y axis = $800/$2 = 400

Equilibrium consumption bundle, via the demand equations:

$$X = \frac{0.25 * 800}{2} = 100$$

$$Y = \frac{0.75 * 800}{2} = 300$$

Question 1b

New price of X after adjusting for the subsidy:

Px = $2 - $1 = $1

New intercepts of the budget constraint:

X axis = $800/$1 = 800

Y axis = $800/$2 = 400 (unchanged)

New consumption bundle, via the demand equations:

$$X = \frac{0.25 * 800}{1} = 200$$

$$Y = \frac{0.75 * 800}{2} = 300 \text{ (unchanged)}$$

Calculating the utility associated with the initial consumption bundle:

$$U1 = (100)^{0.25} * (300)^{0.75} = 227.95$$

Inserting this and the new prices into the expenditure function:

$$M3 = 227.95 * \left(\frac{1}{0.25}\right)^{0.25} \left(\frac{2}{0.75}\right)^{0.75} = \$672.72$$

Calculating the CV:

CV = M3 – M1 = $673 - $800 = -$127

Since the CV is negative, the household is better off under the policy. However, the government's expenditure on the subsidy is $1*200 = $200, which is greater than the gain to the household. The difference between the cost of the subsidy, $200, and the gain to households, $127, is deadweight loss. That is, the policy costs the government $73 more than it produces in benefits ($200-$127=$73).

Question 2

The PV of benefits is the sum of the PV of the extra property taxes (years 1-30) and the extra sales taxes (years 1+):

$$PVb = PVprop + PVsales$$

Computing the components:

$$PVprop = \frac{\$15M}{0.05} - \frac{\left(\frac{\$15M}{0.05}\right)}{1.05^{30}} = \$300M - \$69.4M = \$230.6M$$

$$PVsales = \frac{\$5M}{0.05} = \$100M$$

Adding:

$$PVb = PVprop + PVsales = \$230.6M + \$100M = \$330.6M$$

The PV of the cost is just the $300M construction cost that would have to be paid in year zero. Subtracting that from the PV of benefits gives the NPV: $330.6M - $300M = $30.6M. Since the NPV is positive, it would be a good idea to proceed with the project: it generates enough benefits that it is possible to completely cover all of its costs and still come out ahead.

Question 3a

The PV of the costs in years 1-10 can be computed as follows:

$$PVc = \frac{\$30M}{0.05} - \frac{\left(\frac{\$30M}{0.05}\right)}{1.05^{10}} = \$600M - \$368.3M = \$231.7M$$

The PV of the benefits, which begin in year 11, can be computed as shown:

$$PVb = \frac{\left(\frac{\$20M}{0.05}\right)}{1.05^{10}} = \$245.6M$$

The NPV is the difference:

NPV = $245.6M - $231.7M = $13.9M

Since the NPV is positive, it would be efficient to undertake the project: the benefits it will produce are large enough that everyone who suffers a cost could be compensated, and some net benefits would still remain.

Question 3b

The first step is to derive the demand equations from the information given plus the usual budget constraint:

$$Y = 2 * X$$

$$M = Px * X + Py * Y$$

$$M = Px * X + Py * 2 * X = (Px + 2 * Py) * X$$

$$X = \frac{M}{Px + 2 * Py}$$

$$Y = \frac{2 * M}{Px + 2 * Py}$$

Using these to find the initial equilibrium:

$$X = \frac{\$130M}{\$10 + 2*\$5} = 6.5M$$

$$Y = \frac{2*\$130M}{\$10 + 2*\$5} = 13M$$

To check whether the tax will raise enough revenue, the first step is to compute the new value of X using the demand equation and the higher price of X, Px = $16:

$$X = \frac{\$130M}{\$16 + 2*\$5} = 5M$$

The revenue raised will be the tax times the new value of X: $6*5M = $30M. The tax does, indeed, generate the desired amount of revenue.

To compute the CV, it is necessary to determine the income, M3, needed to obtain the original utility at the new prices. Since the household regards the goods as perfect complements, M3 will have to be large enough for it to be able to buy the original consumption bundle:

M3 = Px*X + Py*Y

M3 = $16*6.5M + $5*13M = $169M

The CV can now be computed:

CV = M3 – M1 = $169M - $130M = $39M

The CV is the true cost to the household of the $30M in tax revenue. It's $9M greater than the tax revenue due to deadweight loss. That is, the household pays $30M to the government and also gives up an additional $9M in surplus it used to get on the 1.5M units of good X that it no longer buy.

Using $39M as the cost of the policy in years 1-10 gives a higher PV cost:

$$PVc = \frac{\$39M}{0.05} - \frac{\left(\frac{\$39M}{0.05}\right)}{1.05^{10}} = \$301.1M$$

Taking this into account, the NPV of the project is actually negative:

NPV = $245.6M - $301.1M = -$55.5M

After accounting for the DWL caused by the tax, the project is no longer a good idea. It does not generate enough benefits to cover all of its costs.