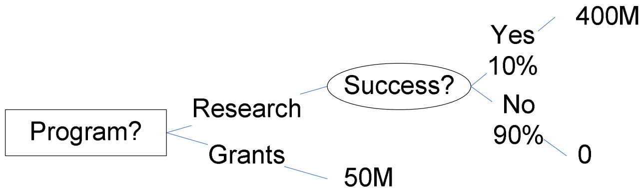
Exam 3, Fall 2006

Notes on Solution

Question 1

Part (a)

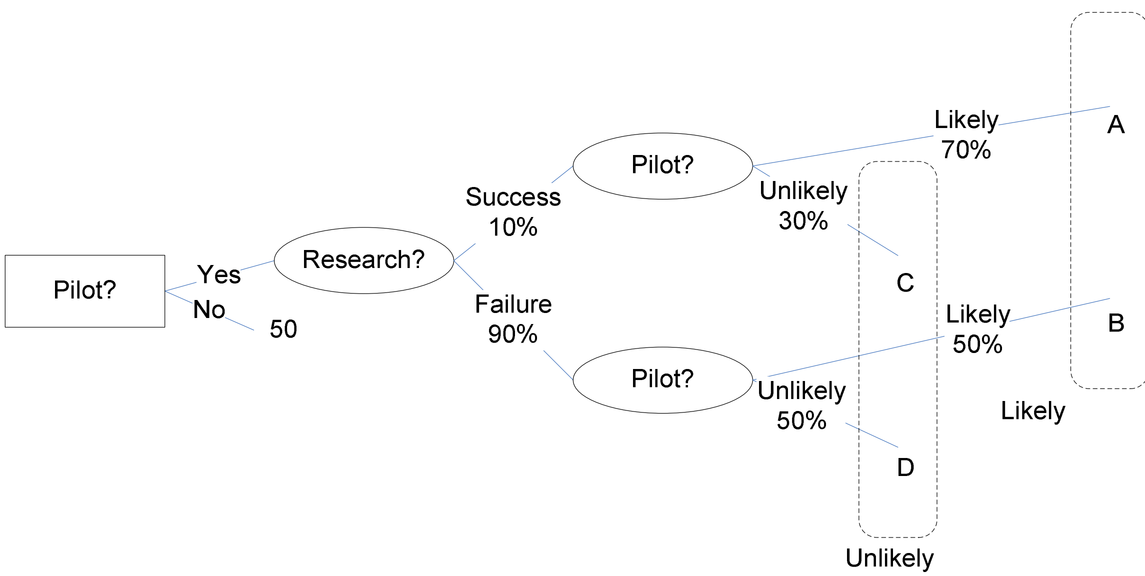
The foundation's decision tree looks as follows (gross payoffs):



The EV of the research project is 0.1*(400M) + 0.9*(0) = 40M. Since that's lower than the alternative, the foundation should give the money out in grants.

Part (b)

The first part of the foundation's decision tree looks as follows:



The probability of ending up at points A-D are listed below:

Point	Research	Fed Prob	Pilot	Prob	Overall
A	Success	10.0%	Likely	70.0%	7.0%
B			Unlikely	30.0%	3.0%
C	Failure	90.0%	Likely	50.0%	45.0%
D			Unlikely	50.0%	45.0%

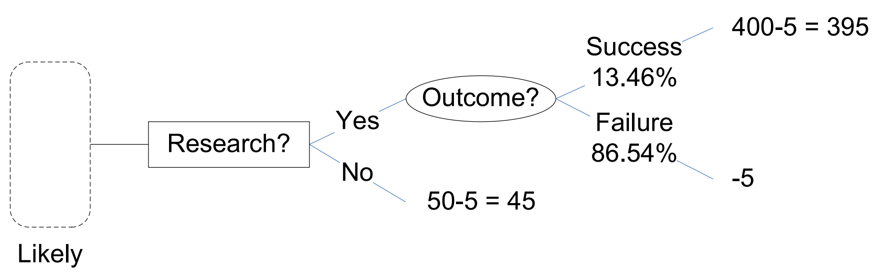
The overall chances of outcome from the pilot study:

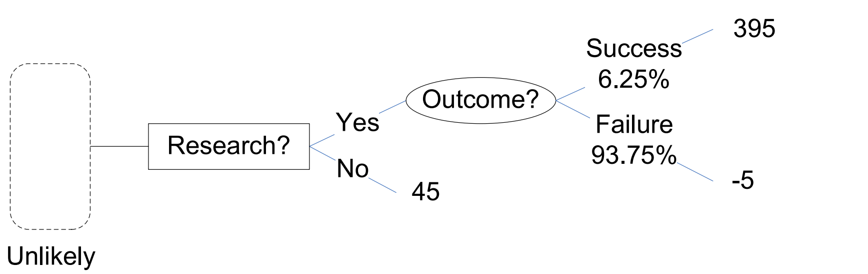
Likely	7% + 45% = 52%
Unlikely	3% + 45% = 48%

Depending on the outcome from the pilot study, the conditional probabilities of the research project succeeding or failing are:

Pilot	Research	Conditional Probability
Likely	Success	7%/52% = 13.46%
	Failure	45%/52% = 86.54%
Unlikely	Success	3%/48% = 6.25%
	Failure	45%/48% = 93.75%

The decision trees branching off from each of the dotted ellipses above are as shown:

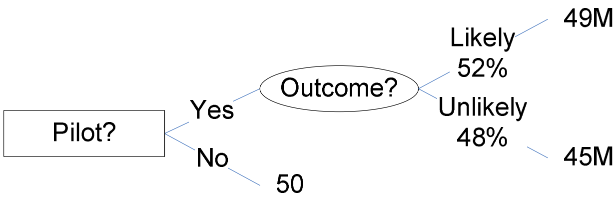




The EV of proceeding with the research project when the pilot indicates that it is likely to succeed is 0.1346*(395M) + 0.8654*(-5M) = 49M. Since that's higher than 45M, if the pilot indicates the project is likely to succeed, the foundation should proceed with the research.

The EV of proceeding with the the research project when the pilot indicates that is is unlikely to succeed is 0.0625*(395M) + 0.9375*(-5M) = 20M. Since that's lower than 45M, if the foundation should not do the research project if the pilot indicates "unlikely".

Finally, taking these outcomes into account, the expected value of the pilot can be calculated:



Even without computing the EV, it's clear that the foundation is better off giving the money out as grants: the payoff is higher than either of the expected payoffs associated with the pilot. Computing the EV just for completeness: EV = 0.52*(49M) + 0.48*(45M) = 47M.

Question 2

Q = K^(0.4) * L^(0.6)

Q	20
Pk	15
Pl	25

	K	L	Q	TC	AC
	15	24.23	20	830.71	41.54
	16	23.21	20	820.20	41.01
	17	22.29	20	812.22	40.61
	18	21.46	20	806.38	40.32
	19	20.70	20	802.39	40.12
	20	20.00	20	800.00	40.00
	21	19.36	20	799.00	39.95
	22	18.77	20	799.22	39.96
	23	18.22	20	800.52	40.03
	24	17.71	20	802.77	40.14
	25	17.24	20	805.89	40.29
Equation		L = (Q/(K^0.4))^(1/0.6)	Q = K^(0.4) * L^(0.6)	TC = Pk*K + Pl*L	AC = TC/Q

The firm should use 21 units of capital and 19.36 units of labor. Its average cost will be $39.95 per unit of output.

Question 3

TC = F + G*Q
P = A - B*Q

F	1340	A	450
G	10	B	10

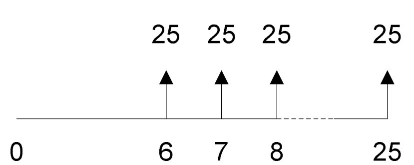
Q	P	TC	TR	Profit	MC	MR
15	300	1490	4500	3010	na	na
16	290	1500	4640	3140	10.00	140
17	280	1510	4760	3250	10.00	120
18	270	1520	4860	3340	10.00	100
19	260	1530	4940	3410	10.00	80
20	250	1540	5000	3460	10.00	60
21	240	1550	5040	3490	10.00	40
22	230	1560	5060	3500	10.00	20
23	220	1570	5060	3490	10.00	0
24	210	1580	5040	3460	10.00	-20
25	200	1590	5000	3410	10.00	-40
	P = 450 - 10*Q	TC = 1340+10*Q	TR = P*Q	Profit = TR-TC	MC = ΔTC/ΔQ	MC = ΔTR/ΔQ

The organization should charge $230 and produce 22 tours. It will earn $3500 in profits.

Question 4

Part (a)

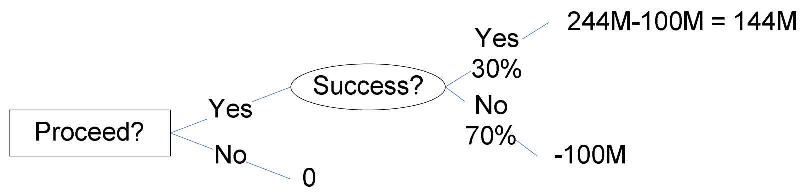
If the project succeeds, each year the firm will produce 5 million units and sell them for P=30-5=25. Total revenue will be $25*5 million = $125 million. Total costs will be $20*5 million = $100 million. Each year's profits, therefore, will be $25 million. The cash stream of cash flows would be:



Computing the PV:

Interest rate:	5%	
PV of an infinite stream:	500	
PV of an infinite stream starting at 6:	391.76	= 500/(1+r)^5
PV of an infinite stream starting at 26:	147.65	= 500/(1+r)^25
PV of stream from 6-25:	244.11	= 391.76 - 147.65

The decision tree for the project is thus:



The EV = 0.3*(144M) + 0.7*(-100M) = -27M. A risk neutral firm would not undertake it: on average, it would expect to lose 27M.

Part (b)

Next task is compute the CS during the patent period (years 6-25):

CS during patent:	12.5	=(1/2)*(30-25)*5
CS if forever	250	=12.5/r
CS from 6 to 25 viewed at year 5	156	=250 - 250/(1+r)^20
CS from 6 to 25 viewed at year 0	122	=156/(1+r)^5

After the patent period, competition in the market will drive the price down to \$20. We can find Q via the demand curve: 20 = 30 - Q, so Q = 10 million.

Q	10	=(30-20)/1
CS after patent	50	=(1/2)*(30-20)*10
CS if forever	1,000	=50/r
Post-patent CS	295	=1000/(1+r)^25

Final step is to add the CS values together. The total CS is the PV of the CS during the patent period plus the PV of the CS after the patent expires:

Total CS:	417	=122+295

Finally, calculating the EV of the CS:

Probability of success:	30%	
EV of CS	125	=0.3*(417) + 0.7*(0)

Part (c)

The subsidy increases both private payoffs by $40 million. Hence the firm's problem becomes the following:

Outcome	Prob	Orig Payoff	New Payoff	Prob*New
Succeeds	30%	144	184	55
Fails	70%	-100	-60	-42
			Expected Profit:	13

The EV is now positive, so the firm will undertake the project.

The CV cost of the $40 million is $40*1.5 = $60 million. Taking this into account the expected net benefit to the public (CS less CV) will be:

Outcome	Prob	CS Payoff	CV cost	Net	Prob*Net
Succeeds	30%	417	60	357	107
Fails	70%	0	60	-60	-42
				Net expected value:	65

Since the EV of the project (including the full CV cost of the revenue needed to underwrite it) is positive, a risk-neutral government would proceed with the subsidy. The overall social surplus, including the returns to the firm, will be 65M + 13M = 78M.