

**Exam 1**  
Spring 2008  
Version M Solution

**Question 1 (Q3 on V; Q5 on Y) (15 points)**

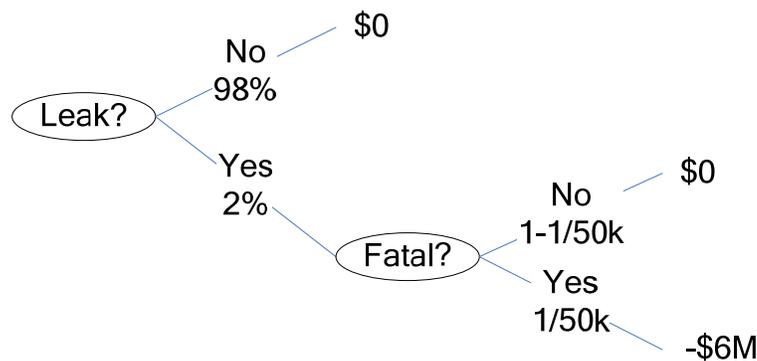
It is technically possible to capture carbon dioxide emissions from coal-fired power plants and store them in old oil reservoirs and other geologic formations. However, there is a chance that the CO<sub>2</sub> might leak out catastrophically. If it does, any nearby residents would potentially be at risk.

Suppose that storing a ton of CO<sub>2</sub> today (year 0) would provide \$5 worth of benefits every year forever from reduced climate change (beginning in year 1). However, suppose that there is a 2% chance in any given year that some of the CO<sub>2</sub> would leak out. If a leak occurs, it would probably dissipate before causing any harm but there is a 1 in 50,000 chance that it would kill someone (CO<sub>2</sub> is non-toxic but in very large volume it could cause asphyxiation).

Using an interest rate of 5% and a VSL of \$6 million, please calculate: (a) the net present value of storing a ton of CO<sub>2</sub> in year 0. On the basis of your results, (b) explain briefly whether or not the project should be carried out. Be sure to show all your work.

Present value of benefits from storing CO<sub>2</sub>:  $\$5/0.05 = \$100$

Expected loss in each year:



Expected cost *if* a leak occurs in any given year (right node):

$$EV \text{ leak} = (1/50,000)*(\$6M) + (49,999/50,000)*(0) = \$120$$

Expected annual cost, taking into account the chance of a leak (left node)

$$EV \text{ cost} = 0.02*(\$120) + 0.98*(0) = \$2.4$$

Present value of expected cost:  $\$2.4/0.05 = \$48$

Net present value:  $\$100 - \$48 = \$52$

[a] NPV = \$52

[b] The project should proceed ahead. The benefits it provides are more than enough to cover the fair insurance premiums that would be needed to compensate for the additional risk.

**Question 2 (Q2 on V; Q1 on Y) (15 points)**

Consumption of a particular good has been found to create a positive externality. The market willingness to pay for the good is  $W2P = 1200 - 2*Q$  and the marginal cost of producing it is  $MC = 200 + 3*Q$ . However, each unit also creates \$100 worth of external benefits.

Please compute: (a), (b) the price and quantity at the market equilibrium, (c) the efficient quantity, and (d) the net welfare gain from moving from the market equilibrium to efficiency. Please note that you only have to calculate the two equilibriums and the efficiency gain: you do not have to propose or discuss any policies in this question.

Finding the market equilibrium:

$$P = W2P = MC$$

$$1200 - 2*Q = 200 + 3*Q$$

$$1000 = 5*Q$$

$$Q = 200$$

$$P = 1200 - 2*200 = 1200 - 400 = \$800$$

$$\text{Check: } MC = 200 + 3*200 = 200 + 600 = 800$$

$$\text{[a] } P = \$800$$

$$\text{[b] } Q = 200$$

Finding the efficient quantity:

$$MSB = W2P + MB_{ext}$$

$$MSB = 1200 - 2*Q + 100 = 1300 - 2*Q$$

$$MSB = MC$$

$$1300 - 2*Q = 200 + 3*Q$$

$$1100 = 5*Q$$

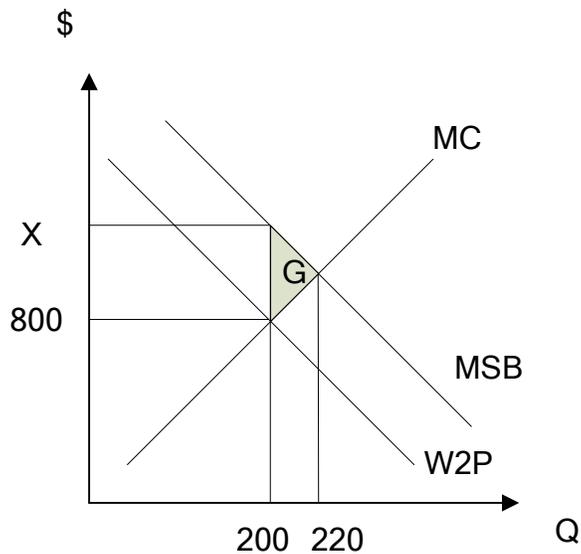
$$Q = 220$$

$$\text{Check: } MSB = 1300 - 2*220 = 1300 - 440 = \$860$$

$$\text{Check: } MC = 200 + 3*220 = 200 + 660 = \$860$$

$$\text{[c] Efficient } Q = 220$$

The welfare gain is triangle G in the diagram below, where X will need to be calculated:



Solving for X, the MSB at  $Q=200$ :

$$X = 1300 - 2 \cdot 200 = 1300 - 400 = \$900 \text{ (the original } \$800 \text{ plus the } \$100 \text{ externality).}$$

$$\text{Area G} = (1/2) \cdot (\$900 - \$800) \cdot (220 - 200) = \$1000$$

[d] Gain is \$1000

### Question 3 (Q5 on V; Q4 on Y) (15 points)

A pollutant is emitted by two different *types* of sources. There are 5 sources of type “A” and 100 sources of type “B”. Each type-A source has a marginal abatement cost curve given by the equation  $MCA_i = 1 \cdot Q_i$  where  $Q_i$  is the amount of abatement done by source  $i$  and  $i$  is a subscript running from 1 to 5. Each type-B source has an abatement cost curve given by  $MCA_j = 20 \cdot Q_j$  where  $j$  is a subscript running from 1 to 100. The marginal benefit of abatement is known to be:  $MBA = 400 - 0.1 \cdot Q_t$ , where  $Q_t$  is total abatement.

Please calculate: (a) the efficient total amount of abatement, (b) the efficient marginal cost of abatement, (c) the efficient amount of abatement done by an *individual* type-A source, and (d) the efficient amount of abatement done by an *individual* type-B source. Note that you only have to find the efficient pattern of abatement: you do not need to discuss a policy in this question.

Rearranging each MCA curve to get Q in terms of MCA:

$$\text{Type A: } Q_i = MCA$$

$$\text{Type B: } Q_j = MCA/20$$

Adding up quantities over sources:

$$Q_t = 5 \cdot Q_i + 100 \cdot Q_j$$

$$Q_t = 5 * MCA + 100 * (MCA/20) = 5 * MCA + 5 * MCA = 10 * MCA$$

$$\text{Rewriting: } MCA = Q_t/10 = 0.1 * Q_t$$

$$MBA = MCA$$

$$400 - 0.1 * Q_t = 0.1 * Q_t$$

$$400 = 0.2 * Q_t$$

$$2000 = Q_t$$

$$\text{Check: } MCA = 0.1 * 2000 = 200$$

$$\text{Check: } MBA = 400 - 0.1 * 2000 = 400 - 200 = 200$$

$$[a] Q_t = 2000$$

$$[b] MCA = \$200$$

Efficient abatement by individual sources:

$$[c] Q_i = MCA = 200$$

$$[d] Q_j = MCA/20 = 200/20 = 10$$

$$\text{Check: } 5 * 200 + 100 * 10 = 1000 + 1000 = 2000$$

#### Question 4 (Q1 on V; Q2 on Y) (15 points)

Three sources emit a pollutant and each source initially emits 500 tons. The marginal abatement costs for the three sources are given by:  $MCA_1 = 1 * Q_1$ ,  $MCA_2 = 1 * Q_2$  (identical to source 1), and  $MCA_3 = 2 * Q_3$ . The marginal benefit of abatement is given by  $MBA = 500 - (3/5) * Q_t$ , where  $Q_t$  is total abatement.

Design a tradable permit system that will achieve the efficient amount of abatement while causing source 3 to bear *all* of the overall compliance cost. Please calculate: (a), (b) and (c) the number of permits that should be distributed to each source.

Finding the overall MCA by solving for Q's and summing:

$$Q_1 = MCA$$

$$Q_2 = MCA$$

$$Q_3 = MCA/2$$

$$Q_t = Q_1 + Q_2 + Q_3 = MCA + MCA + MCA/2 = (5/2) * MCA \text{ or } 2.5 * MCA$$

$$\text{Rearranging: } MCA = (2/5) * Q_t$$

Finding the efficient amount of abatement:

$$MBA = MCA$$

$$500 - (3/5) * Q_t = (2/5) * Q_t$$

$$500 = (5/5) * Q_t = Q_t$$

Checking:  $MBA = 500 - (3/5)*500 = 500 - 300 = 200$

Checking:  $MCA = (2/5)*500 = 200$

Efficient pattern of abatement:

$Q1 = MCA = 200$

$Q2 = MCA = 200$

$Q3 = MCA/2 = 200/2 = 100$

Checking:  $200 + 200 + 100 = 500 = Q_t$

Permits each source will need to hold in the end:

Source 1:  $500 - 200 = 300$

Source 2:  $500 - 200 = 300$

Source 3:  $500 - 100 = 400$

Checking:  $300 + 300 + 400 = 1000 = 1500 - Q_t$

Total abatement cost for each source will be  $(1/2)*MCA*Q_i$ :

$Cost1 = (1/2)*(\$200)*200 = \$20,000$

$Cost2 = (1/2)*(\$200)*200 = \$20,000$

$Cost3 = (1/2)*(\$200)*100 = \$10,000$

In equilibrium, the price of a permit will equal the efficient MCA, or \$200. To shift all costs to source 3, the permits will have to be allocated so that source 3 buys \$20,000 worth of permits from each of sources 1 and 2. Translating that into permits:

$\$20,000/\$200 = 100$  permits

The initial allocation should thus be:

Source 1:  $300 + 100 = 400$

Source 2:  $300 + 100 = 400$

Source 3:  $400 - 200 = 200$

Checking:  $400 + 400 + 200 = 1000$

[a] Source 1: 400 permits

[b] Source 2: 400 permits

[c] Source 3: 200 permits

### Question 5 (Q4 on V; Q3 on Y) (15 points)

Two sources of a pollutant were recently regulated. Just before regulation, each source was emitting 100 tons of the pollutant (200 tons total). The MBA for the pollutant is \$200 per ton for levels of abatement up to 80 tons; above 80 tons of abatement the MBA drops to \$50 per ton. At the time of regulation, the sources were believed to be able to abate at the following costs:

$MCA1 = 2*Q1$ ,  $MCA2 = 2*Q2$ . An emissions tax policy was established and the tax set at \$80 per ton. However, the projected MCA for source 1 turned out to be wrong. The true curve was  $MCA1 = 4*Q1$ .

Please calculate: (a), (b) the amount of abatement that would have been done by each firm if the original MCA1 had been correct; (c), (d) the amount of abatement that would be efficient for each firm given the true MCA1; (e), (f) the actual amount of abatement done by each firm; and (g) the deadweight loss due to the policy.

Under the \$80 tax, both firms abate until their MCA equals the tax:

$$Q1 = MCA/2 = 80/2 = 40$$

$$Q2 = MCA/2 = 80/2 = 40$$

$$[a] Q1 = 40$$

$$[b] Q2 = 40$$

Given the true MCA2, the overall MCA curve would be:

$$Q1 = MCA/2$$

$$Q2 = MCA/4$$

$$Qt = Q1 + Q2 = MCA/2 + MCA/4$$

$$Qt = (3/4)*MCA$$

$$MCA = (4/3)*Qt$$

It will still be efficient to clean up to the 80 ton threshold as long as the MCA at 80 is between \$50 and \$200. Checking:

$$MCA \text{ at } 80 = (4/3)*80 = 106.67 \text{ or } 106 \frac{2}{3}$$

Yes, it's still efficient to hit the threshold. The efficient pattern of abatement:

$$Q1 = MCA/2 = 106.67/2 = 53.33$$

$$Q2 = MCA/4 = 106.67/4 = 26.67$$

$$\text{Checking: } 53.33 + 26.67 = 80$$

$$[c] Q1 = 53.33$$

$$[d] Q2 = 26.67$$

The actual abatement will be determined by each firm's response to the \$80 tax:

$$Q1 = MCA/2 = 80/2 = 40$$

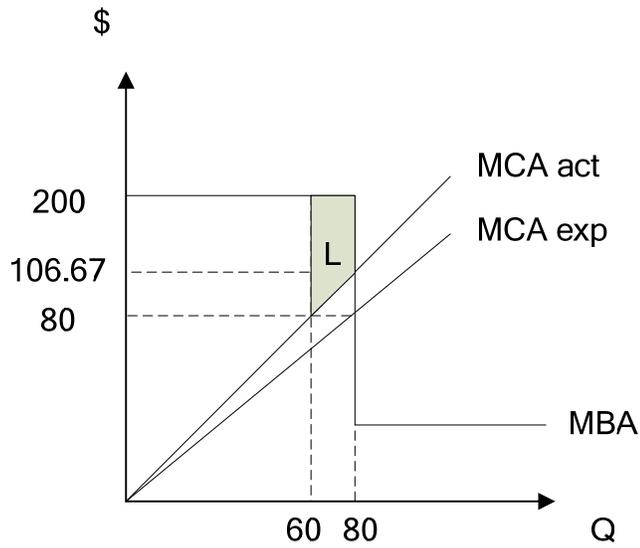
$$Q2 = MCA/4 = 80/4 = 20$$

$$[e] Q1 = 40$$

$$[f] Q2 = 20$$

Total abatement will be 60 tons, 20 tons below the efficient amount. The deadweight loss will be the area between the MBA curve and the actual MCA curve. It's area L in

the diagram below:



Computing it:

$$L = (80-60)*(200-106.67) + (1/2)*(80-60)*(106.67-80)$$

$$L = \$1,866.60 + \$266.70 = \$2,133.30$$

$$[g] \text{ DWL} = \$2,133$$