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## Exam 2

Fall 2008

## DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO.

## Instructions

Write your SUID in the upper right corner of this exam. Do NOT write your name.
SHOW ALL YOUR WORK. Answers without supporting work will receive little or no credit.

Do all your work on this exam. If you need extra space, write on the backs of the pages. However, if you do write an answer on the back of a page, be sure you've noted that near the question.

There are 70 points on the exam and you'll have 80 minutes to work on it. Budget your time accordingly.

## Data on Households

Some of the questions will refer to the following survey data collected from several household about their consumption of goods X and Y over the last couple of years.

Table 1

|  | Year | Income | Px | Py | Qx | Qy |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: |
| Household R | 2006 | 100 | 1 | 2 | 60 | 20 |
|  | 2007 | 112 | 2 | 2 | 42 | 14 |
|  | 2006 | 100 | 1 | 2 | 60 | 20 |
|  | 2007 | 120 | 2 | 2 | 36 | 24 |
| Household T | 2006 | 90 | 1 | 2 | 60 | 15 |
|  | 2007 | 120 | 2 | 2 | 30 | 30 |

## Part 1: Perfect Complements (30 points total)

## Question 1a (10 points)

As you know, a household that regards two goods, X and Y , as perfect complements likes to have exactly $a$ units of the X good for each unit of Y. Please derive the household's demand equations for X and Y in terms of $a, \mathrm{Px}, \mathrm{Py}$ and income M . Be sure to show the steps involved, don't just write down the equations.

Next, use the demand equations and the information in Table 1 to determine which of the surveyed households has perfect complements preferences and calculate the appropriate value of $a$.

## Question 1b (10 points)

Now suppose that in 2008, $\mathrm{Px}=\$ 2$, $\mathrm{Py}=\$ 1$ and the household from 1a has an income of $\$ 140$. Calculate the household's consumption of each good. Draw the budget constraint and include the numerical values of its intercepts. Also sketch several of its indifference curves and show the household's equilibrium on the diagram. Be sure to show your work and label everything.

## Question 1c (10 points)

Suppose that good X is made from crude oil and a sudden shock to the world oil market causes the 2008 value of Px to jump to $\$ 3$. The price of good Y remains $\$ 1$. Please calculate the new X and Y for the household from 1b. Then calculate the household's compensating variation. What is the CV as a percent of the household's 2008 income?

## Part 2: Cobb-Douglas (30 points total)

## Question 2a (10 points)

A household whose preferences can be represented by a Cobb-Douglas utility function of the form $U=X^{g} Y^{1-\mathrm{g}}$ has demand equations $\mathrm{X}=g^{*} \mathrm{M} / \mathrm{Px}$ and $\mathrm{Y}=(1-g)^{*} \mathrm{M} /$ Py where $g$ is an unknown parameter and M is income.

Use the demand equations and the information in Table 1 to determine which of the surveyed households has Cobb-Douglas preferences and calculate the value of $g$ for that household. Then derive the household's expenditure function.

## Question 2b (10 points)

Suppose that in 2008, $\mathrm{Px}=\$ 2$, $\mathrm{Py}=\$ 1$ and the household from 2a has an income of $\$ 200$. Calculate the household's consumption of each good. Draw the household's budget constraint and include the numerical values of its intercepts. Also sketch several of its indifference curves and show its equilibrium on the diagram. Be sure to show your work and label everything.

## Question 2c (10 points)

Suppose the government wishes to cut the household's consumption of Y in half (that is, reduce the Y from part 2 b by 50 percent) by adding a tax that would raise Py. Please calculate what Py would have to be to achieve the goal. How much revenue will the tax raise? (You may assume that the supply of Y is perfectly elastic so that the difference between the new price and the original Py of $\$ 1$ will be exactly equal to the tax.) How large is the compensating variation? What is the CV as a percent of the household's 2008 income?

## Part 3: Other Preferences (10 points total)

When none of the utility functions we've discussed in class matches a household's preferences very well the following function is sometimes used:

$$
\mathrm{U}=\left(\mathrm{X}^{0.5}+\mathrm{Y}^{0.5}\right)^{2}
$$

The corresponding demand equations and expenditure function are:

$$
X=\frac{M^{* P y}}{P x^{*}(P x+P y)}, \quad Y=\frac{M^{*} P x}{P y^{*}(P x+P y)}, \quad M=U *\left(\frac{P x * P y}{P x+P y}\right)
$$

Suppose that a household with these preferences has $\$ 50$ of income. Initially $\mathrm{Px}=1$ and $\mathrm{Py}=1$ but the government imposes a $\$ 1$ tax on good X and its price rises to $\mathrm{Px}=2$. Please calculate the amount of tax revenue that will be generated. What is the CV for the policy and how does it compare to the revenue?

Finally, given the change in X caused by the policy, how would you expect the ICs for these preferences to compare to Cobb-Douglas: flatter (more like perfect substitutes) or sharper (more like perfect complements)? Explain briefly.

