

Final Exam, Fall 2003

Notes on Solution

Question 1

Production function: $Q = K^{0.5} \cdot L^{0.5}$

Pk 50

Pl 32

Solving for L as a function of Q and K:

$$Q = K^{0.5} \cdot L^{0.5}$$

$$Q/K^{0.5} = L^{0.5}$$

$$(Q/K^{0.5})^2 = L$$

$$Q^2/K = L$$

Q	K	L	Check	TC	AC
20	14	28.57	20	1614.29	80.71
20	15	26.67	20	1603.33	80.17
20	16	25.00	20	1600.00	80.00
20	17	23.53	20	1602.94	80.15
20	18	22.22	20	1611.11	80.56
20	19	21.05	20	1623.68	81.18
20	20	20.00	20	1640.00	82.00

On the exam you could actually stop calculating after this line because costs are now rising. The K and L mix has moved past the point of cost minimization.

Use 16 units of K and 25 units of L. Average costs will be \$80 per unit.

Question 2

Costs: $100 + 4 \cdot Q^2$

Demand: $P = A - B \cdot Q$

A 390

B 5

Q	P	TR	MR	TC	AC	MC	MR-MC	AR-AC	Profit
40	190	7600		6500	162.5			27.5	1100
41	185	7585	-15	6824	166.4	324	-339	18.6	761
42	180	7560	-25	7156	170.4	332	-357	9.6	404
43	175	7525	-35	7496	174.3	340	-375	0.7	29
44	170	7480	-45	7844	178.3	348	-393	-8.3	-364
45	165	7425	-55	8200	182.2	356	-411	-17.2	-775
46	160	7360	-65	8564	186.2	364	-429	-26.2	-1204
47	155	7285	-75	8936	190.1	372	-447	-35.1	-1651
48	150	7200	-85	9316	194.1	380	-465	-44.1	-2116
49	145	7105	-95	9704	198.0	388	-483	-53.0	-2599
50	140	7000	-105	10100	202.0	396	-501	-62.0	-3100

On the exam you could stop calculating after the Q=44 line because that's where losses begin to occur. The other problems on the exam are similar: it isn't necessary to compute the entire table; you can stop once you can tell you've gone beyond the optimum point.

Charge \$175 and serve 43 clients. Will earn a very small profit of \$29.

Question 3

$$Q = 3K + L^{1/2}$$

$$Q - 3K = L^{1/2}$$

$$(Q - 3K)^2 = L$$

Pk 15
 PI 22
 P 220

Part (a)

Q	K	L	P	TR	MR	TC	AC	MC	MR-MC	Profit
15	4	9	220	3300		258	17.20		0	3042
16	4	16	220	3520	220	412	25.75	154	66	3108
17	4	25	220	3740	220	610	35.88	198	22	3130
18	4	36	220	3960	220	852	47.33	242	-22	3108
19	4	49	220	4180	220	1138	59.89	286	-66	3042
20	4	64	220	4400	220	1468	73.40	330	-110	2932

The firm should hire 25 units of labor and produce 17 units of output.

Part (b)

Profit = 3130

Output will be approximately efficient because each firm will be producing where its MC is as close as possible to P; thus W2P for the last unit will be almost equal to MC. In the long run, the profits will cause other firms to enter the industry, expanding output and driving prices down until profits are driven to zero.

Question 4

Demand: $P = A - B \cdot Q$
 A 1255
 B 2
 MC 25
 Test Cost 600,000
 Prob Good 20%

Part (a)

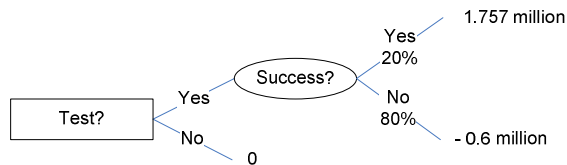
Optimal annual P, Q and profits if testing succeeds

Q	P	TR	MR	MC	MR-MC	Profit
305	645	196,725		25		189,100
306	643	196,758	33	25	8	189,108
307	641	196,787	29	25	4	189,112
308	639	196,812	25	25	0	189,112
309	637	196,833	21	25	-4	189,108
310	635	196,850	17	25	-8	189,100
311	633	196,863	13	25	-12	189,088
312	631	196,872	9	25	-16	189,072
313	629	196,877	5	25	-20	189,052
314	627	196,878	1	25	-24	189,028
315	625	196,875	-3	25	-28	189,000

Part (b)

PV if the stream lasted forever:	3,782,240	=	189,112	/	0.05
PV of losing stream after 20 years:	1,425,486	=	3,782,240	/	1.05 ²⁰
	1,425,486	=	3,782,240	/	2.653298
Difference:	2,356,754				
Net value after paying for the test:	1,756,754	=	2,356,754	-	600,000

Decision tree:



EV of testing:	0.20	*	1,756,754	+	0.80	*	-600,000
EV of testing:	-128,649						

Firm should not conduct the test because the test's EV is negative: on average, the firm would lose money on projects like this.

Question 5

Part (a)

If testing is successful, efficient price would be MC, which is \$25

Finding the level of competitive output requires rearranging the demand curve:

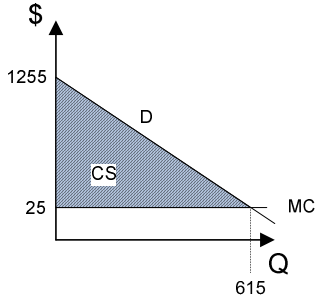
$$\begin{aligned}
 P &= A - B \cdot Q \\
 P - A &= -B \cdot Q \\
 A - P &= B \cdot Q \\
 (A - P) / B &= Q
 \end{aligned}$$

$$\begin{aligned}
 P &= 25 \\
 Q &= 615
 \end{aligned}$$

The market will arrive at this price because at a higher price, firms in the market would be earning profits and new firms would enter. That would raise output and lower the price. At a lower price, firms would be losing money and some would leave. That would reduce output and raise the price.

Part (b)

Value of the drug would be the associated CS, which is given by the area of the triangle below. There is no PS because the price the firms receive exactly matches their costs of production.



$$CS = \frac{(1255 - 25) \times 615}{2}$$

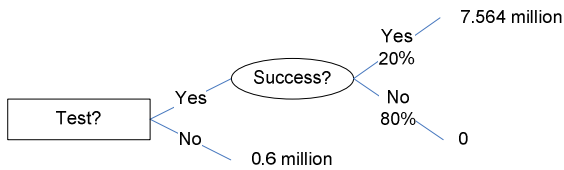
$$CS = 378,225 \text{ per year}$$

Part (c)

If the NGO goes ahead with the test and the test succeeds, the drug will become available forever starting in year 1.

$$PV \text{ of receiving CS forever} = \frac{378,225}{0.05} = 7,564,500$$

Decision tree using gross payoffs and taking into account that if the NGO doesn't buy the test it can deliver \$600,000 of benefits via cash grants:



$$\begin{aligned} \text{Gross EV of testing:} & \quad 0.20 \times 7,564,500 + 0.80 \times 0 \\ \text{EV of testing:} & \quad 1,512,900 \\ \text{EV of cash grants:} & \quad 600,000 \end{aligned}$$

Would prefer to test the drug; testing generates extra expected benefits equal to: 912,900

Part (d)

The firm's decision is not efficient: the NGO calculation shows that testing has an expected value of 1,512,900 while it only costs 600,000. Not testing is thus inefficient: the patients who would gain from the drug would gain enough to compensate the firm for the cost of testing and would still come out ahead.

If the firm had gone ahead anyway, the situation still would not have been efficient because during the patent period it would have kept P too high and Q too low (Q would be half of the efficient amount: 308 instead of 615). Once the patent expired, the price would drop and the market would move to efficiency.

Lengthening the patent period would increase the PV of the firm's monopoly profits and would increase the chance that it tests the potential drug. By itself, that's good. However, a longer patent life would increase the inefficiency due to underprovision of the drug by the monopolist. Shortening the patent life would reduce the monopoly problem but would also reduce the incentive for testing the drug.

This dilemma is typical of pharmaceuticals: we need to offer patents (and hence monopoly profits) in order to give private firms an incentive to develop and test new drugs. However, once the drugs have been developed, for efficiency we'd strongly prefer to provide them to as many people as possible by selling them at MC. There is, unfortunately, no easy way out of the dilemma.