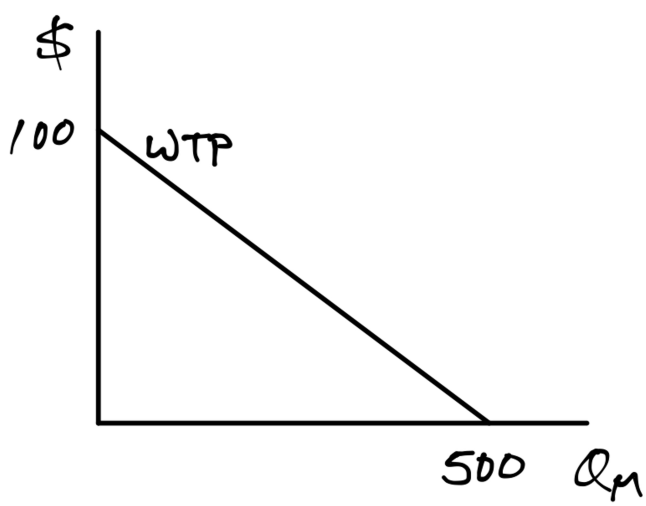
Second Example of a Positive Externality

Initial information:

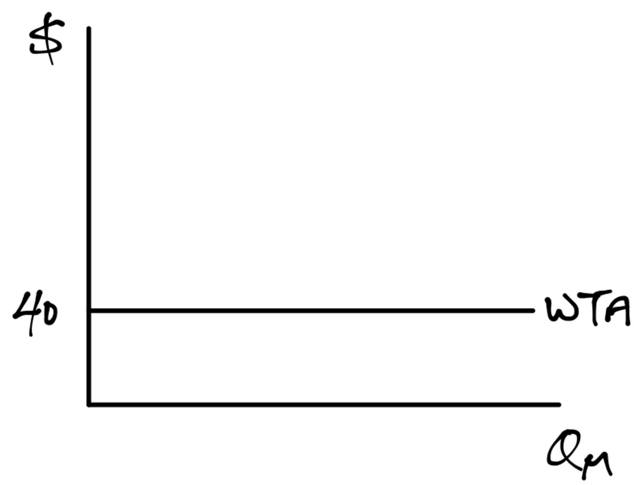
$$WTP = 100 - \frac{1}{5} Q_M^D$$
$$WTA = 40$$
$$MB_e = \frac{1}{20} Q_M^D$$

Graphing each piece:

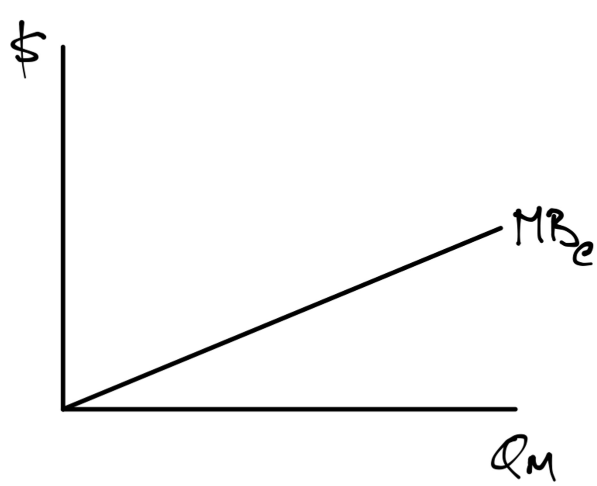
Demand, WTP



Supply, WTA



Externality, MB_e



Solving for the market equilibrium:

$$WTP = WTA$$

$$100 - \frac{1}{5}Q_M^D = 40$$

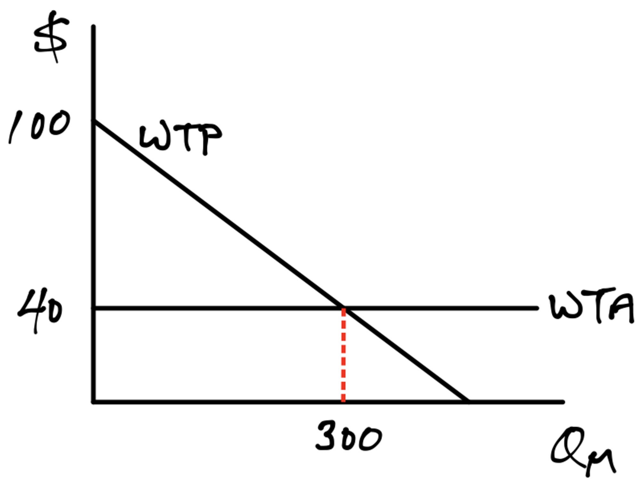
$$Q_M^D = 300$$

Externality on unit 300:

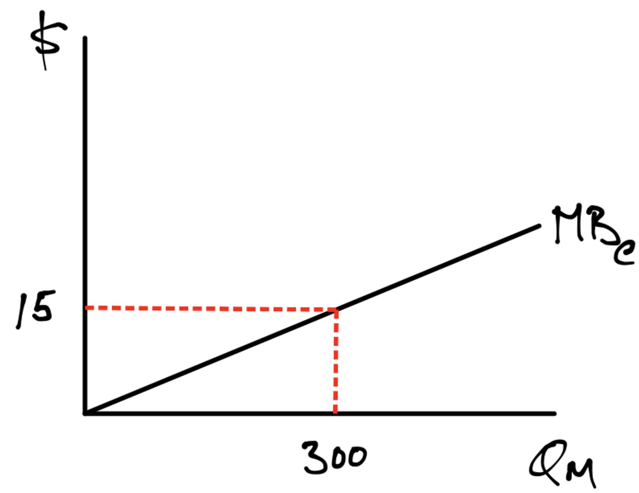
$$MB_e = \frac{1}{20}(300) = \$15$$

Graphing it:

Market



Externality

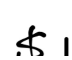


Now solve for the efficient equilibrium:

$$MSB = 100 - \frac{1}{5}Q_M^D + \frac{1}{20}Q_M^D$$

$$MSB = 100 - \frac{3}{20}Q_M^D$$

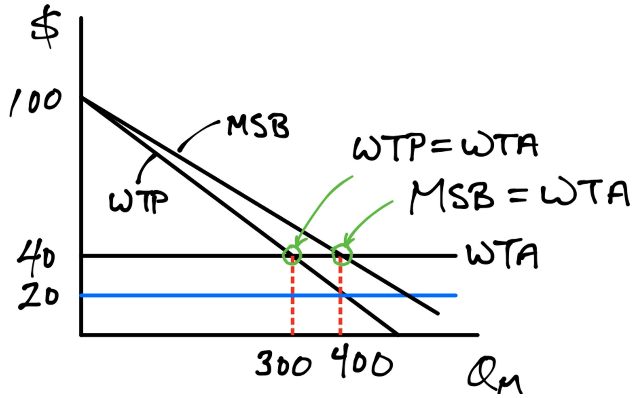
$$MSB = WTA$$



$$MSB = WTA$$

$$100 - \frac{3}{20} Q_M^D = 40$$

$$Q_M^D = \frac{20}{3} * 60$$
$$Q_M^D = 400$$



Calculating the subsidy needed:

Want: $Q_M^D = 400$

P_2^d needed:

$$P_2^d = WTP$$
$$P_2^d = 100 - \frac{1}{5} Q_M^D$$
$$P_2^d = 100 - \frac{1}{5} * 400 = \$20$$

P_2^s needed:

$$P_2^s = WTA$$

$$P_2^s = \$40$$

Subsidy needed:

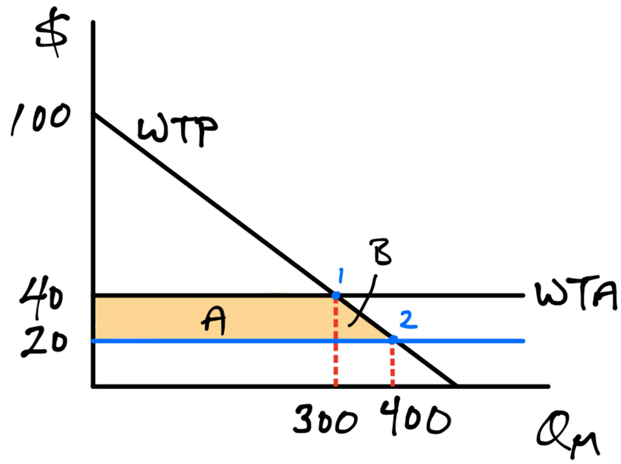
$$S = P_2^s - P_2^d = \$40 - \$20 = \$20$$

Check:

$$S = MB_e(Q_M^e) = \frac{1}{20} * 400 = \$20 \checkmark$$

Impacts on welfare:

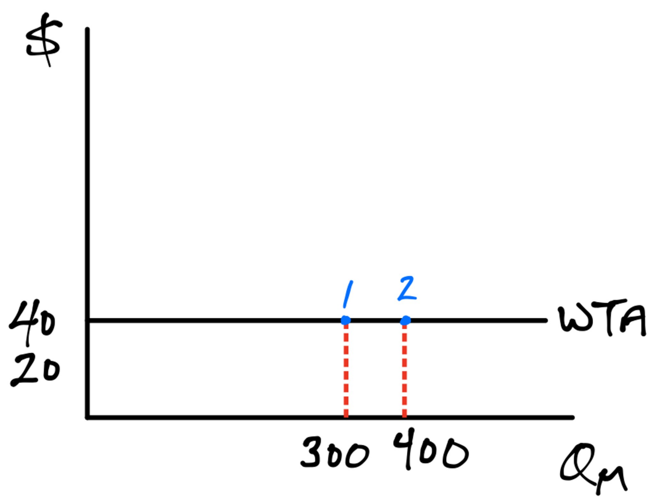
Buyers:



A = 20*300 = 6,000
B = 0.5*20*100 = 1,000

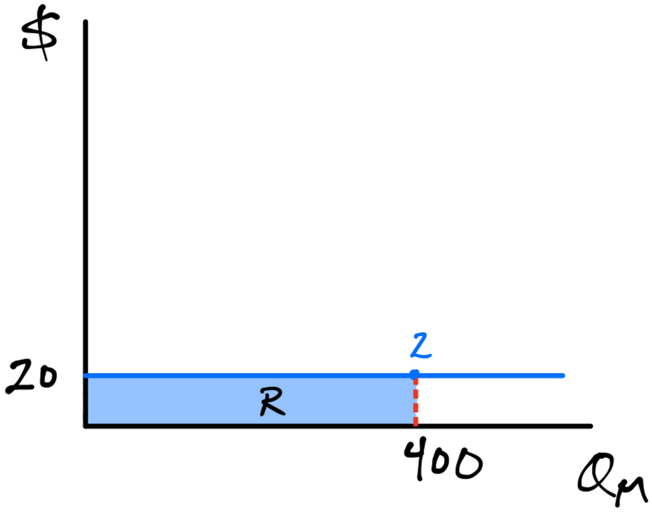
$\Delta CS = +7{,}000$

Sellers:



$\Delta PS = 0$

Government:

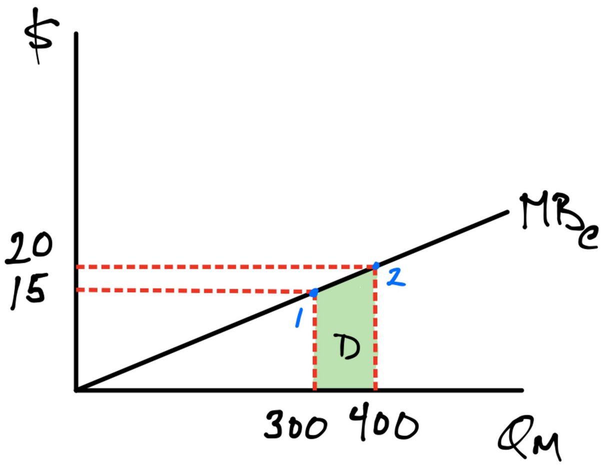


$R = 20*400 = 8{,}000$

$\Delta Rev = -R$

$\Delta Rev = -8{,}000$

Externality:



D = $\frac{15+20}{2} * 100 = 1750$

$\Delta Ext = +D$

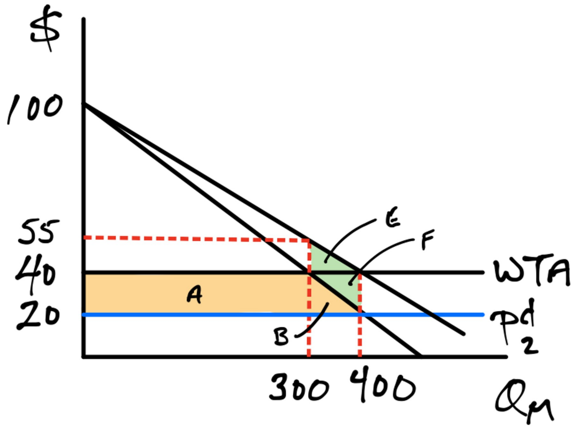
$\Delta Ext = +1{,}750$

Overall impact on SS:

Buyers:	+7,000
Sellers:	0
Government:	-8,000
Externality:	+1,750

$\Delta SS = +750$

Combining into a single diagram:



$\Delta SS = E$

E = 0.5*15*100 = 750

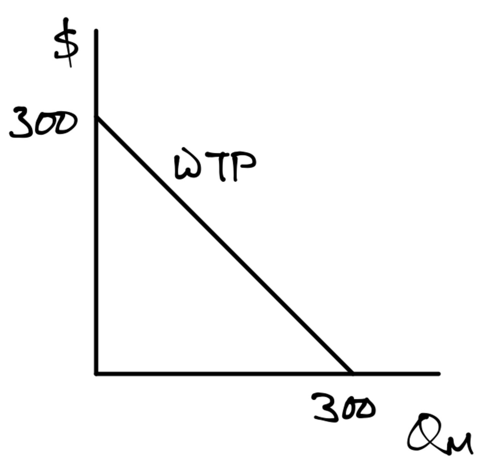
Negative Externality

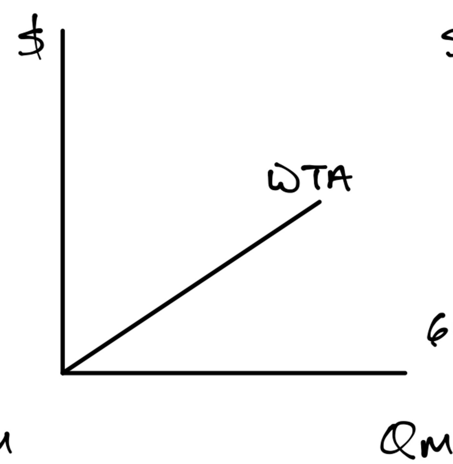
Exactly reverse of positive externalities:

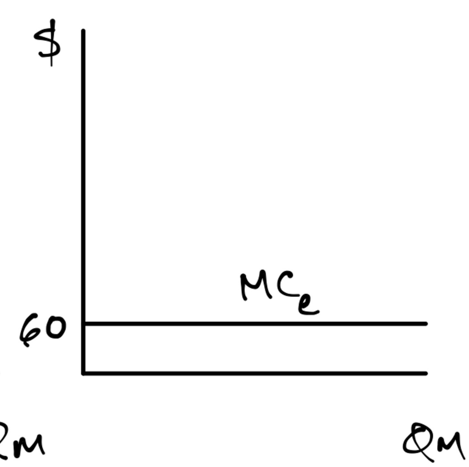
	Positive	Negative
Impact on third parties:	Benefit	Cost
Algebraic form:	MB_e	MC_e
Market Q:	Too small	Too large
Marginal social B or C:	$MSB = WTP + MB_e$	$MSC = WTA + MC_e$
Needed for efficiency:	$MSB = WTA$	$WTP = MSC$
Corrective policy:	$S = MB_e$	$T = MC_e$

Example:

$$WTP = 300 - Q_M^D$$
$$WTA = 2Q_M^S$$
$$MC_e = 60$$



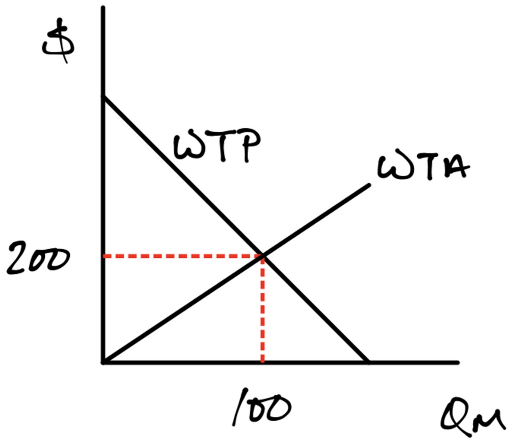




Market equilibrium:

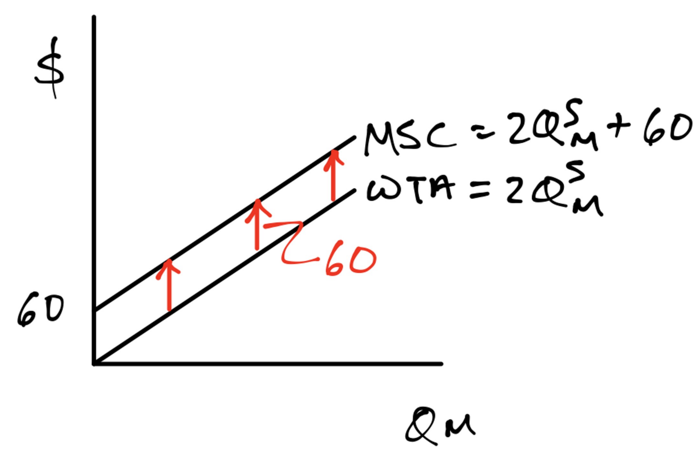
$$WTP = WTA$$
$$300 - Q_M^D = 2Q_M^S$$
$$300 = 3Q_M^D$$
$$Q_M^D = 100$$

$$P_1^d = WTP = 300 - 100 = \$200$$
$$P_1^s = WTA = 2(100) = \$200$$



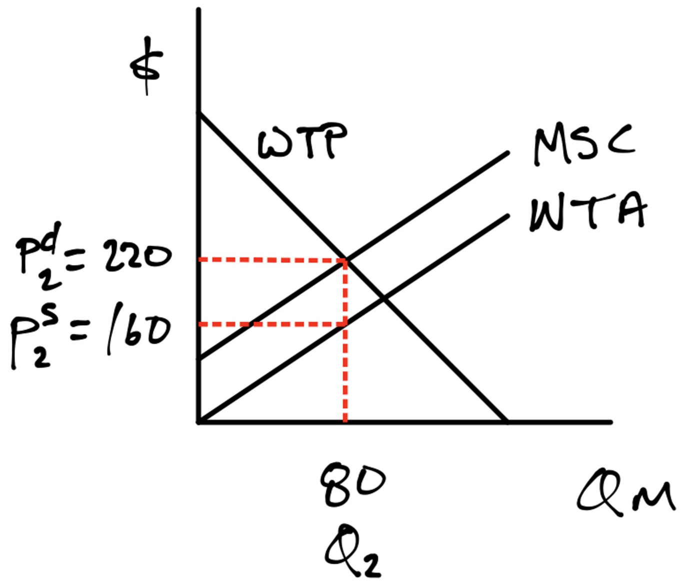
Efficient equilibrium:

$$MSC = 2Q_M^S + 60$$



$$WTP = MSC$$
$$300 - Q_M^D = 2Q_M^S + 60$$
$$240 = 3Q_M^D$$
$$Q_M^D = 80$$

$$P_2^d = WTP = 300 - 80 = 220$$
$$P_2^s = WTA = 2(80) = 160$$



Policy needed:

$$P_2^d = P_2^s + T$$

$$220 = 160 + T$$
$$T = 60$$

Check: $T = MC_e = 60$ ✓

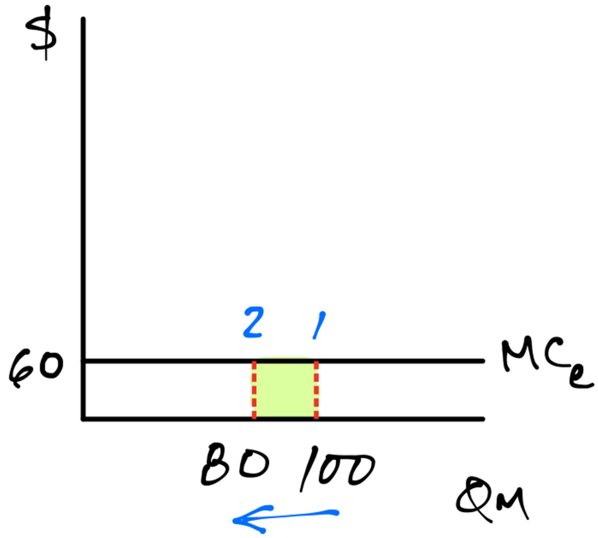
Welfare impacts:

ΔCS, ΔPS, ΔRev calculated as usual:

$$\Delta CS = -\left(\frac{80 + 100}{2}\right) * (220 - 200) = -\$1800$$
$$\Delta PS = \left(\frac{80 + 100}{2}\right) * (160 - 200) = -\$3600$$

$$\Delta Rev = 60 * 80 = +\$4800$$

ΔExt is the area under the MC_e curve between the old and new Q:



$\Delta Ext = 60 * (100 - 80)$
$\Delta Ext = 60 * 20$
$\Delta Ext = \$1200$

Computing ΔSS:

ΔCS	-$1800
ΔPS	-$3600
ΔRev	+$4800
ΔExt	+$1200

$\Delta SS = +\$600$

Cross-Subsidies

Definition:

A cross-subsidy is a policy that provides a subsidy in one market paid for by charging extra in another market.

In effect:
S in one market
T in another market

Usual goal:
Break even over all

Example: Post Office

Market	Cost per Letter
Urban (U)	WTA_U
Rural (R)	WTA_R

Rural is more expensive to serve:
$WTA_R > WTA_U$

Single weighted-average price P charged for both:
U: $P > WTA_U$
R: $P < WTA_R$

In effect, tax in U and subsidy in R:
$T_U = P - WTA_U$
$S_R = WTA_R - P$

Example: Local and long distance telephone service

Prior to deregulation in 1980's:

- One phone company in the US: ATT
- Two markets:
  1. Local lines (L)
  2. Long distance minutes (M)
- Prices regulated
- Allowed to charge extra in M to subsidize L

Analyze a stylized version below

Case 1: BAU

Local market (lines):

Price per month:	$P_{L1} = \$30$
Quantity:	$Q_{L1} = 100\ k$
Demand elasticity:	$\eta_L = -0.2$
Cost to provide:	$WTA_L = ?$
Effective subsidy:	$S = ?$

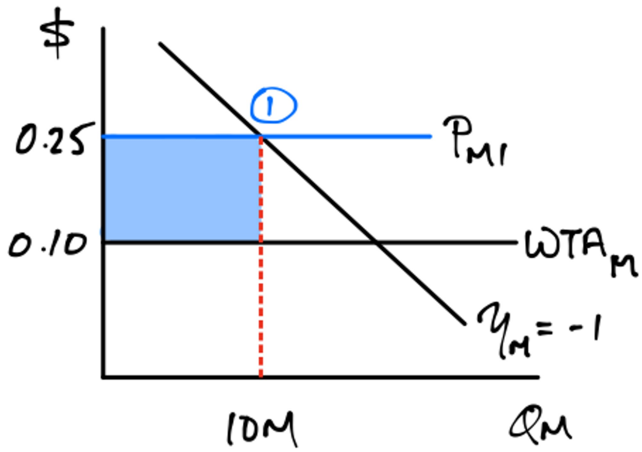
Long distance market (minutes):

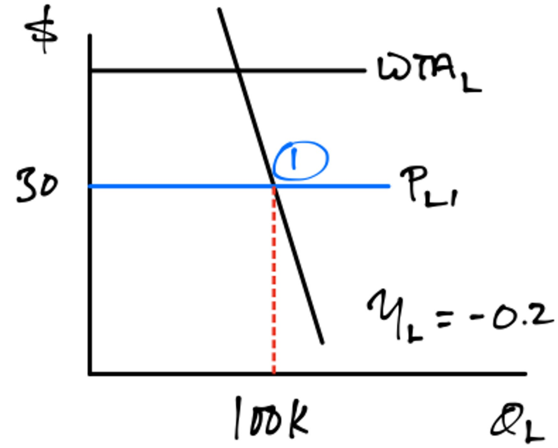
Price per minute:	$P_{M1} = \$0.25$
Quantity:	$Q_{M1} = 10\ M$
Demand elasticity:	$\eta_M = -1$
Cost to provide:	$WTA_M = \$0.10$
Effective tax:	$T = \$0.15$

Budget is balanced:

Tax revenue in M = Subsidy expenditure in L

Graphing:





Determining S and WTA_L:

Tax revenue raised in long distance market (M):

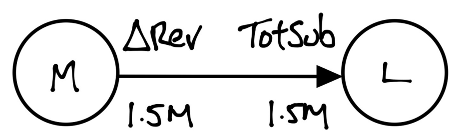
$$\Delta Rev_M = \$0.15 * 10\ M = \$1.5\ M$$

Total subsidy expenditure in local market (L):

$$TotSub_L = S_L * Q_{L1} = S_L * 100\ k$$

Budget balanced:

$$TotSub_L = \Delta Rev_M$$



$$S_L * 100\ k = \$1.5\ M$$

Local subsidy per line:

$$S_L = \frac{\$1.5\ M}{100\ k} = \$15$$

WTA_L:

$$P_{L1} + S = WTA_L$$

$$\$30 + \$15 = \$45 = WTA_L$$

Case 2: Eliminate cross subsidy

Long distance market:

$$P_{M2} = WTA_M = \$0.10$$

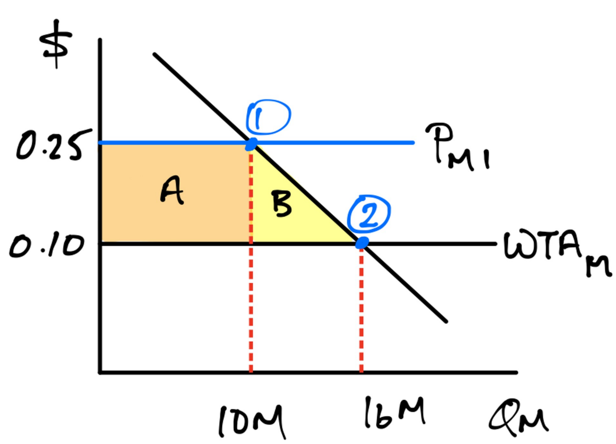
$$\%\Delta P_M = -\frac{0.15}{0.25} = -60\%$$

$$\eta = \frac{\%\Delta Q_M}{\%\Delta P_M}$$

$$-1 = \frac{\%\Delta Q_M}{-60\%}$$

$$\%\Delta Q_M = +60\%$$

$$Q_{M2} = 10M + 0.6 * 10M = 16M$$



A = 0.15*10M = $1.5 M

B = 0.5*0.15*6M = $450 k

$$\Delta CS = +(A + B) = \$1.95\ M$$
$$\Delta Rev = -A = -\$1.5\ M$$

$$\Delta SS_M = +B = \$450\ k$$

Removing the cross subsidy:

Net gain in long distance market

Market for local lines:

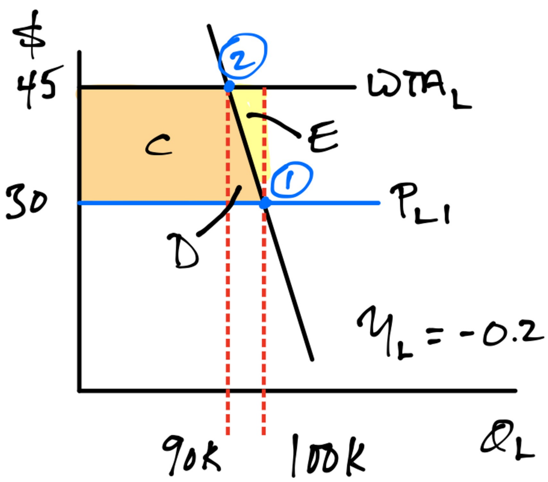
$P_{L2} = WTA_L = \$45$

$$\%\Delta P_L = +\frac{\$15}{\$30} = +50\%$$

$$\%\Delta Q_L = \eta_L * \%\Delta P_L$$

$$\%\Delta Q_L = -0.2 * 50\% = -10\%$$

$$Q_{L2} = 100\ k\ - 0.1 * 100\ k = 90\ k$$



C = 15*90 k = $1.35 M
D = 0.5*15*10 k = $75 k
E = 0.5*15*10 k = $75 k

$\Delta CS = -(C + D) = -\$1.425\ M$
$\Delta Rev = +(C + D + E) = +\$1.5\ M$

$\Delta SS_L = +E = +\$75\ k$

Overall impact on both markets together:

Eliminating the cross subsidy:

Gain in long distance: $450 k
Gain in local: $75 k
Total gain: $525 k

Interpretation:
Net **cost** of **having** the cross subsidy: $525 k

Overall, cross subsidy has two impacts:

Costs $1.950 M to M consumers

Delivers $1.425 M to L consumers