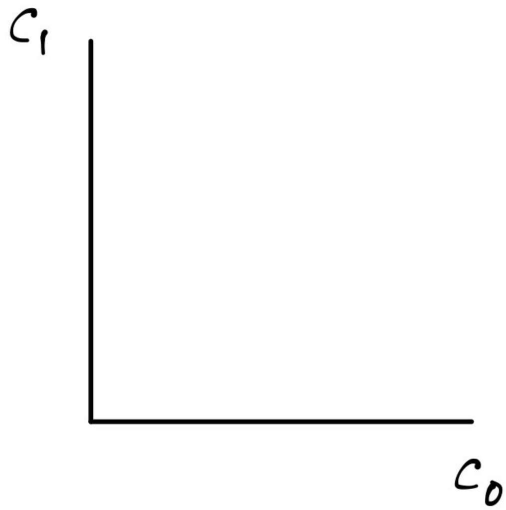
Life Cycle Model of Savings

Choosing bundle of current (C_0) and future consumption (C_1):



Constructing the feasible set:

What bundles of C_0 and C_1 are feasible?

To make concrete, suppose:

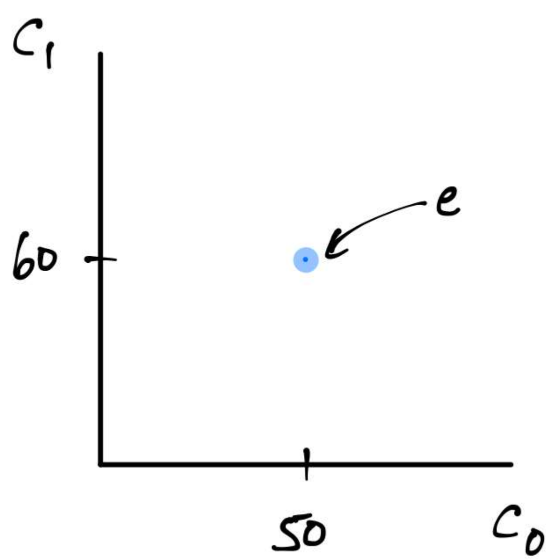
$I_0 = 50{,}000$
$I_1 = 60{,}000$
$r = 10\%$

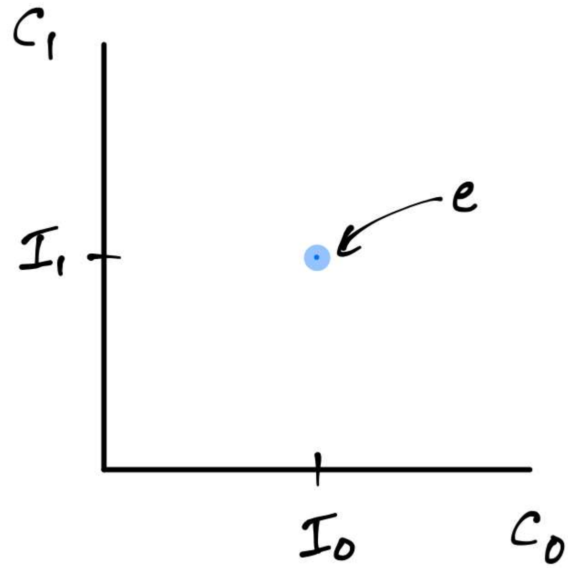
First feasible point:

Consume income when it arrives: no saving or borrowing
Called the endowment point ("e" in figures)

$C_0 = I_0 = 50{,}000$
$C_1 = I_1 = 60{,}000$

This example	General case





Also feasible: bundles reachable by saving

Suppose save in period 0:

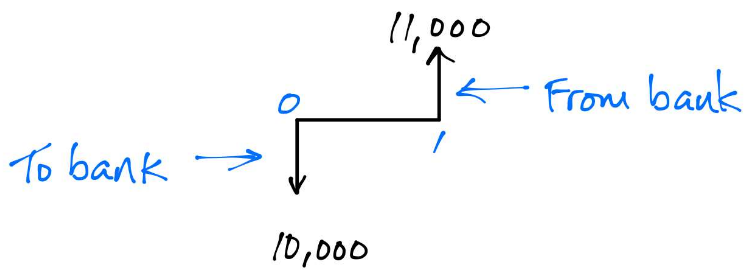
$S = 10{,}000$ to bank

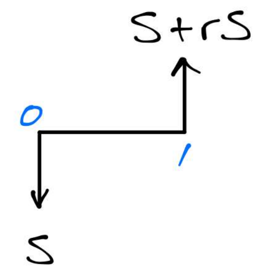
Bank balance in period 1:

Variable	Example amount	General form
Principal:	$\$10{,}000$	S
Interest:	$0.1{*}\$10{,}000 \ = \ \$1{,}000$	rS
Total:	$\$11{,}000$	$S + rS = S(1 + r)$

Cash flow diagram, from saver's perspective:

This example	General

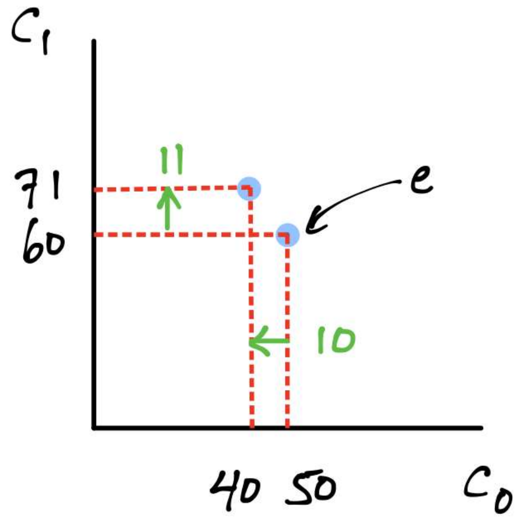


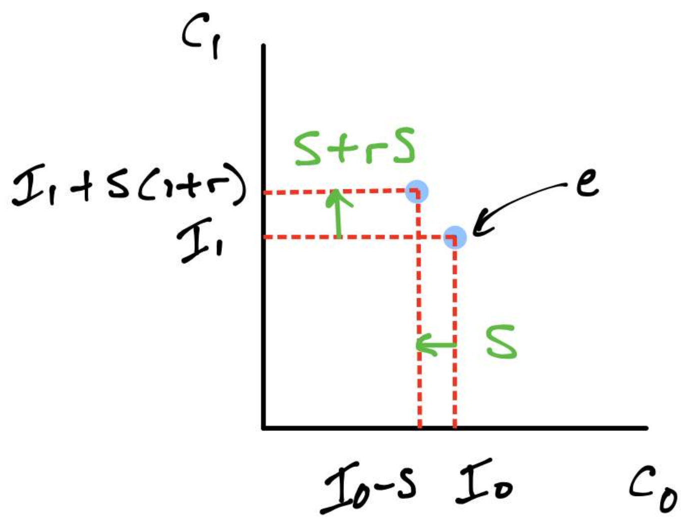


Horizontal axis: timeline
Inflows to agent: point up (positive)
Outflows from agent: point down (negative)

Adding point to the diagram:

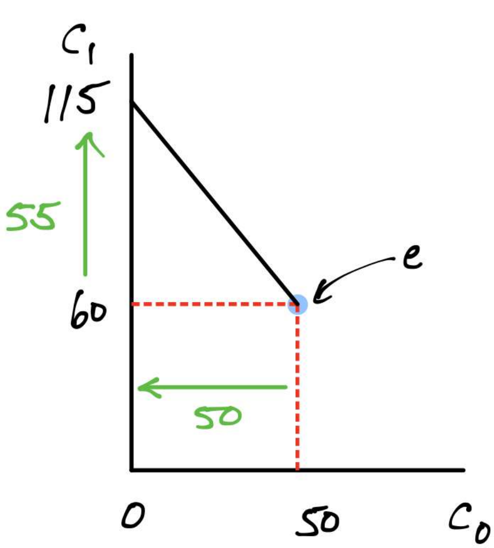
This example	General case

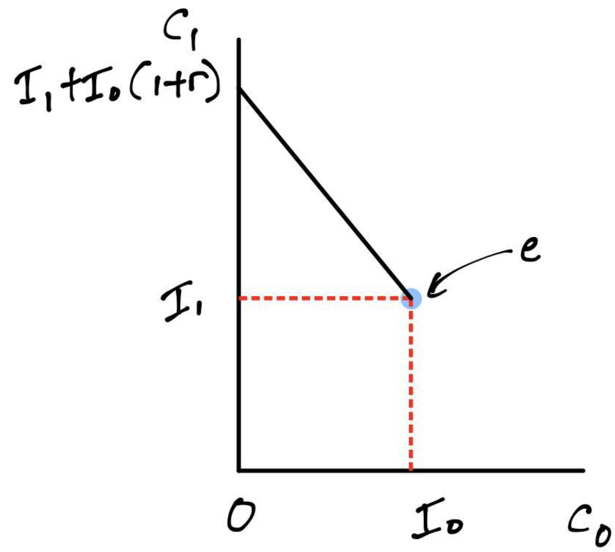




Accessible via other amounts of saving:

This example	General case





Y intercept, future value of income (FVI):

$$FVI = I_1 + I_0(1+r)$$

Slope:

$$\frac{\Delta Y}{\Delta X} = \frac{S(1+r)}{-S} = -(1+r)$$

Bundles feasible by borrowing:

Suppose borrow in period 0:

$B = 10{,}000$ from bank

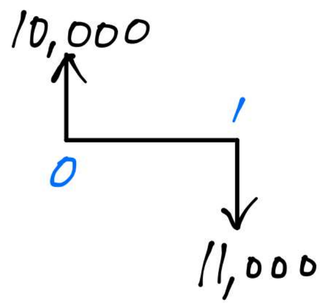
Owed to bank in period 1:

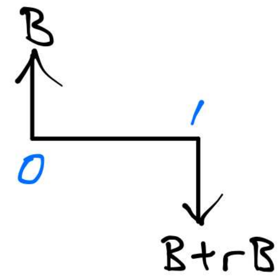
Variable	Example amount	General form
Principal:	$\$10{,}000$	B
Interest:	$0.1*\$10{,}000 = \$1{,}000$	rB

Total:	\$11,000	$B + rB = B(1+r)$

Cash flow diagram:

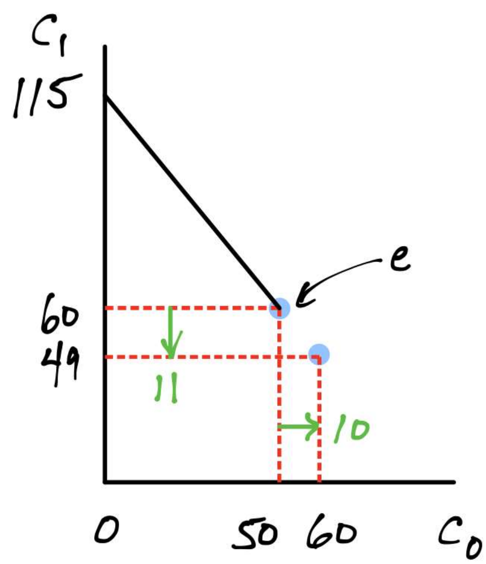
This example	General

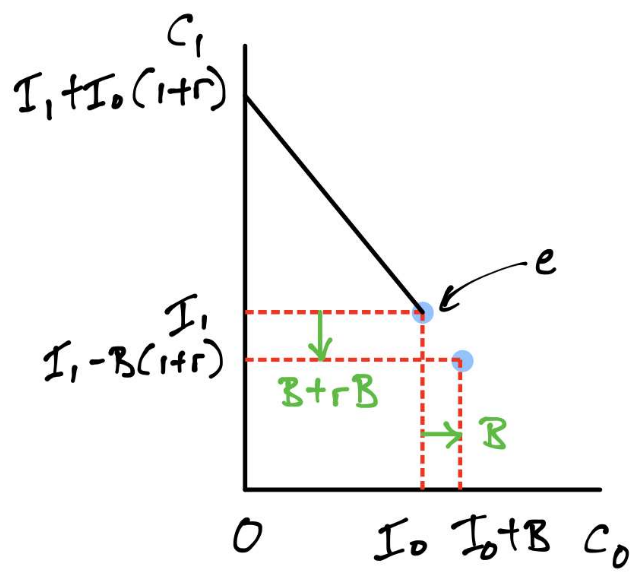




Adding to diagram:

This example	General case





Maximum loan, $\hat{B}$, that can be repaid using I_1:

$$\hat{B} + r\hat{B} = \hat{B}(1+r) = I_1$$

$$\hat{B} = \frac{I_1}{1+r}$$

$$\hat{B} = \frac{60{,}000}{1.1} = 54{,}545$$

Maximum consumption $\hat{C}_0$:

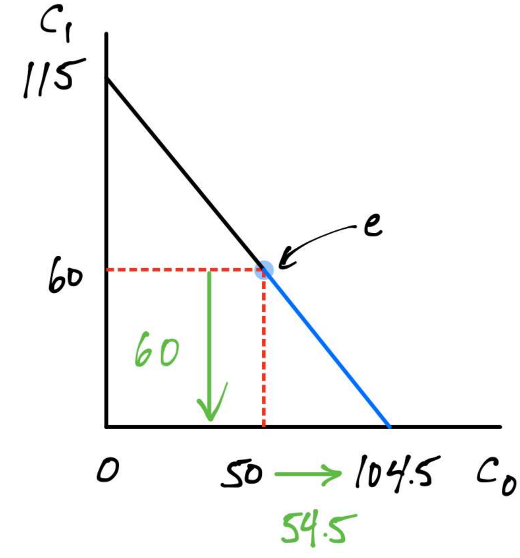
$$\hat{C}_0 = I_0 + \hat{B}$$

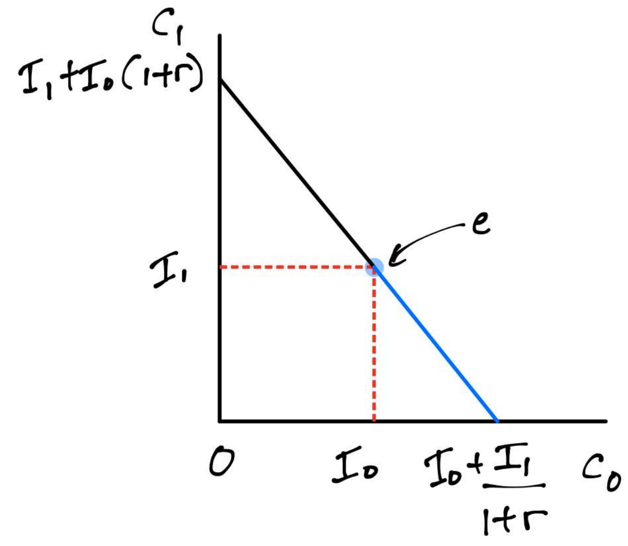
$$\hat{C}_0 = I_0 + \frac{I_1}{1+r}$$

$$\hat{C}_0 = 50{,}000 + \frac{60{,}000}{1.1} = 104{,}545$$

Full range of bundles accessible by borrowing:

This example	General case





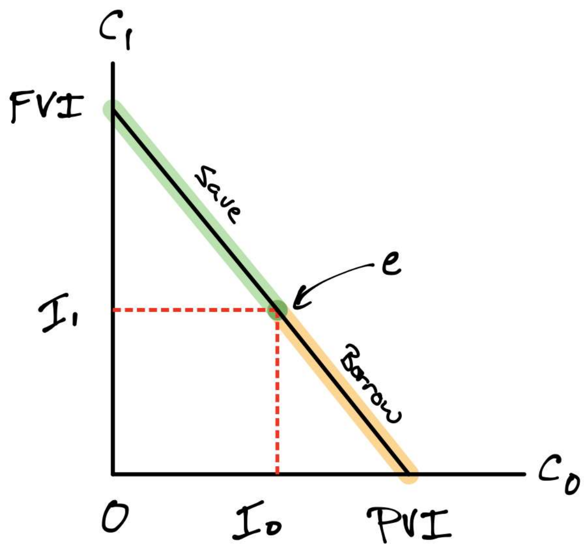
X intercept: present value of income (PVI)

$$PVI = I_0 + \frac{I_1}{1+r}$$

Slope:

$$\frac{\Delta Y}{\Delta X} = \frac{-B(1+r)}{B} = -(1+r)$$

Finished two-period intertemporal BC:



$$FVI = I_1 + I_0(1+r)$$

$$PVI = I_0 + \frac{I_1}{1+r}$$

Daily exercise on Google Classroom