Finding an Equation for the Intertemporal BC

Use $B$ to find feasible combinations of $C_{0}$ and $C_{1}$ (could also use $S$ )
Suppose person borrows $B$ dollars and consumes it at 0

$$
C_{0}=I_{0}+B
$$

Must repay $B(1+r)$ in period 1

$$
C_{1}=I_{1}-B(1+r)
$$

Solving the $C_{1}$ equation for $B$

$$
\begin{aligned}
& C_{1}-I_{1}=-B(1+r) \\
& B=\frac{I_{1}-C_{1}}{1+r}
\end{aligned}
$$

Insert into $C_{0}$ equation to

$$
C_{0}=I_{0}+\frac{I_{1}-C_{1}}{1+r}
$$

Regroup by date of variable:

$$
\left(C_{0}-I_{0}\right)(1+r)=I_{1}-C_{1}
$$

Interpretation: must repay all loans

Regroup by type of variable:
$C_{0}+\frac{C_{1}}{1+r}=I_{0}+\frac{I_{1}}{1+r}$
$C_{0}+\frac{C_{1}}{1+r}=P V I$
Interpretation: present value of $C$ equals present value of $I$

From here on, we'll use this version of the BC :

$$
C_{0}+\frac{C_{1}}{1+r}=P V I
$$

Gives feasible bundles in terms of PVI:


To use, convenient to rewrite it slightly:

$$
1 * C_{0}+\left(\frac{1}{1+r}\right) * C_{1}=P V I
$$

Compare to the previous BC :

$$
P_{x} * Q_{x}+P_{y} * Q_{y}=M
$$

Lining up the terms:

| Two goods | Two periods |
| :--- | :--- |
| $P_{x}$ | $P_{0}=1$ |
| $Q_{x}$ | $C_{0}$ |
| $P_{y}$ | $P_{1}=1 /(1+r)$ |
| $Q_{y}$ | $C_{1}$ |
| $M$ | $P V I$ |

Prices are period O's view when decision is made:
$P_{0}=1 \quad$ Price of $\$ 1$ of period- 0 consumption in 0 $P_{1}=1 /(1+r) \quad$ Price of $\$ 1$ of period -1 consumption at 0

Intuition behind $P_{1}$ :
To get extra \$1 in 1, deposit $S$ in 0 :

$S$ needs to be large enough to grow to $\$ 1$ in period 1:

$$
\begin{aligned}
& S(1+r)=\$ 1 \\
& S=\frac{\$ 1}{1+r}=P_{1}
\end{aligned}
$$

## Example:

$$
\begin{gathered}
r=5 \%=0.05 \\
P_{1}=\frac{1}{1.05}=0.9524 \\
\$ 1 \\
0.9524
\end{gathered}
$$

Applications Page 4

