Example: CD Preferences

First step: build a two-period version of the CD equations

Links between variables from last class:

Two goods	Two periods
P_x	$P_0 = 1$
Q_x	C_0
P_y	$P_1 = 1/(1+r)$
Q_y	C_1
M	PVI

The intertemporal CD utility function is straightforward:

Two goods	Two periods
$U = Q_x^a Q_y^{1-a}$	$U = C_0^a C_1^{1-a}$

The demand equations are a bit more complicated:

Two goods	Two periods
$Q_x = \frac{a * M}{P_x}$	$C_0 = \frac{a * PVI}{P_0}$
$Q_y = \frac{(1-a) * M}{P_y}$	$C_1 = \frac{(1-a) * PVI}{P_1}$

Inserting P_0 and P_1:

$$C_0 = \frac{a * PVI}{1}$$

$$\mathrm{C_1} = \frac{(1 - \mathrm{a}) * \mathrm{PVI}}{\frac{1}{1 + \mathrm{r}}}$$

Can simplify to:

$$C_0 = a * PVI$$

$$C_1 = (1 + r) * (1 - a) * PVI$$

Summarizing the two-period CD functions:

$$U = C_0^a C_1^{1-a}$$

$$C_0 = a * PVI$$

$$C_1 = (1 + r) * (1 - a) * PVI$$

Example problem preferences and income:

$$U = C_0^{\frac{1}{3}} C_1^{\frac{2}{3}}$$

$$I_0 = 30k$$
$$I_1 = 30k$$
$$r = 10\%$$

Computing PVI:

$$PVI = I_0 + \frac{I_1}{1+r}$$

$$PVI = 30k + \frac{30k}{1.1} = 57.3k$$

Demands:

$$C_0 = a * PVI$$

$$C_0 = \left(\frac{1}{3}\right) * 57.3 = 19.1$$

$$C_1 = (1+r) * (1-a) * PVI$$

$$C_1 = 1.1 * \left(\frac{2}{3}\right) * 57.3 = 42.0$$

Save or borrow?

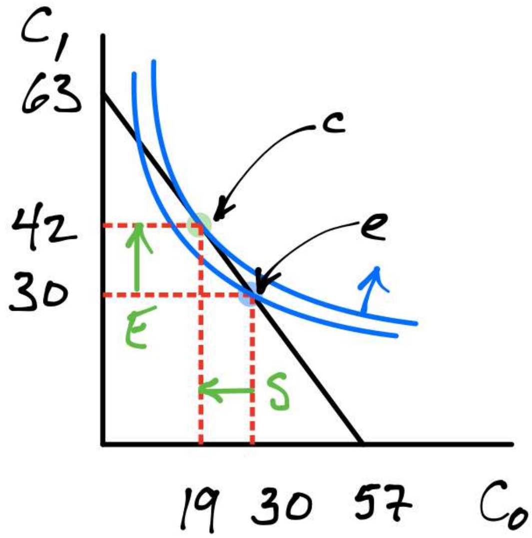
$I_0 = 30k$
$C_0 = 19.1k$

Saves in 0:
$30k - 19.1k = 10.9k$

Earned in 1:
$S(1+r) = 10.9k * 1.1 = 12k$

Check:
$I_1 + S(1+r) = 30k + 12k = 42k = C_1$ ✔

Graphing:


C_1
63
42
30
c
e
E
S
19
30
57
C_0

Human Capital

Investing in education and training:

Spend money now in order to have higher wages in the future

Example:

Income endowment:

$$I_0 = 25k$$
$$I_1 = 25k$$

Can also take classes in period 0 to raise income in period 1:

Define variables:

Tu = tuition paid at 0
Ra = raise in period 1

Suppose the following options are available:

Classes	Tu	Ra
0	0	0
1	5k	10k
2	10k	17k
3	15k	23k
4	20k	28k
5	25k	32k

Each class costs $5k and raises income, but at a decreasing rate

Resulting options for *net income* after accounting for tuition and raise:

$$I_0^{net} = I_0 - Tu$$

$$I_1^{net} = I_1 + Ra$$

In thousands:

Classes		I_0	Tu	I_0^{net}		I_1	Ra	I_1^{net}
0		25	0	25		25	0	25
1		25	5	20		25	10	35
2		25	10	15		25	17	42
3		25	15	10		25	23	48
4		25	20	5		25	28	53
5		25	25	0		25	32	57

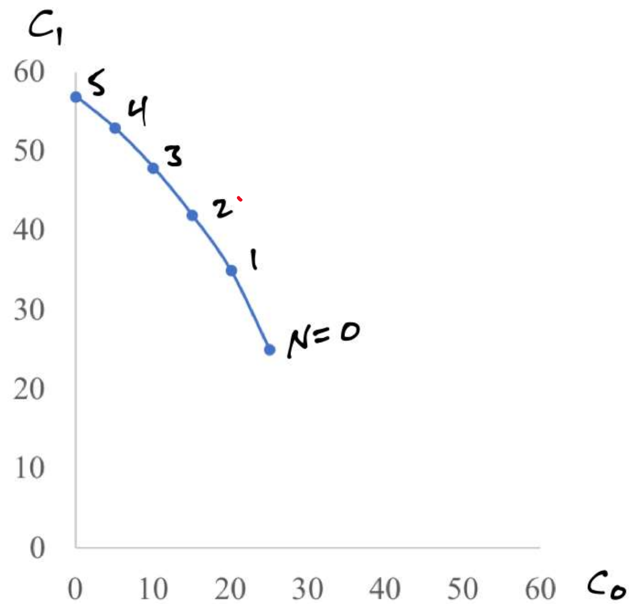
Can choose income bundle by adjusting number of classes

Which number of classes is best?

Initially, suppose can't borrow or save: must consume net income

$$C_0 = I_0^{net}$$
$$C_1 = I_1^{net}$$

Graphing the options:

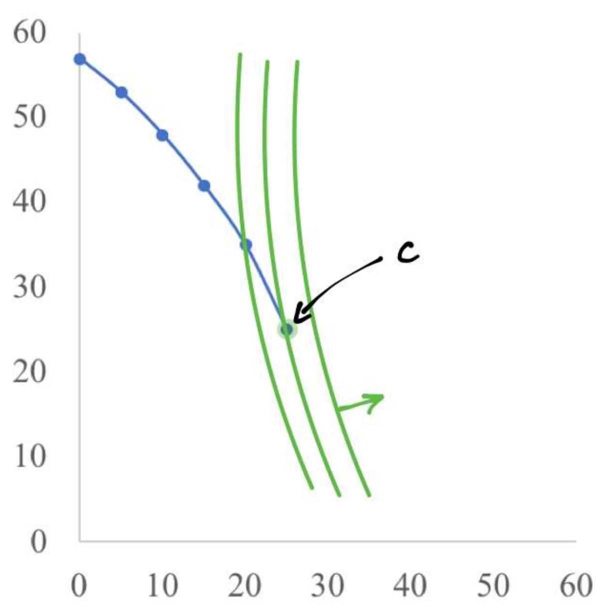


Known as a "human capital production function"

Feasible set of consumption bundles achievable by school alone.

Now add ICs to find the option chosen:

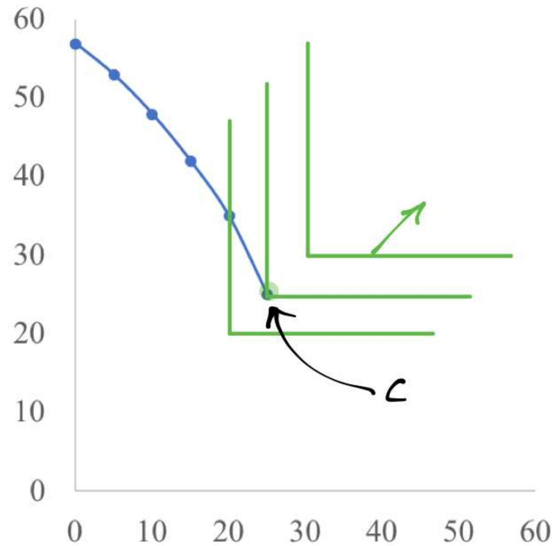
Case 1: Very steep ICs



N=0

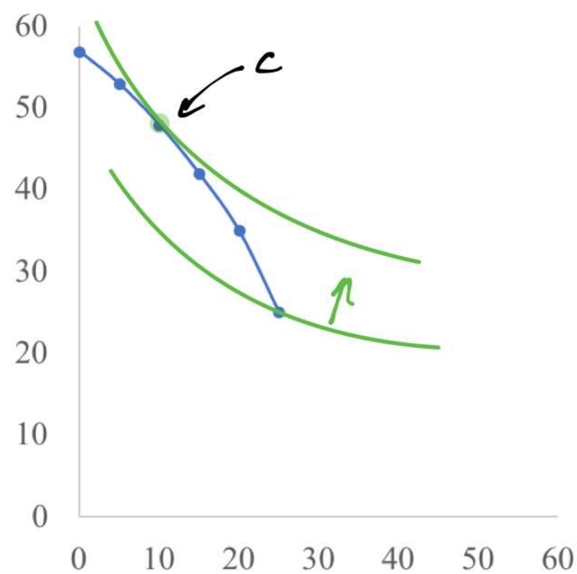
Case 2: Perfect complements

$$\frac{C_0}{C_1} = \frac{1}{1}$$



N=0

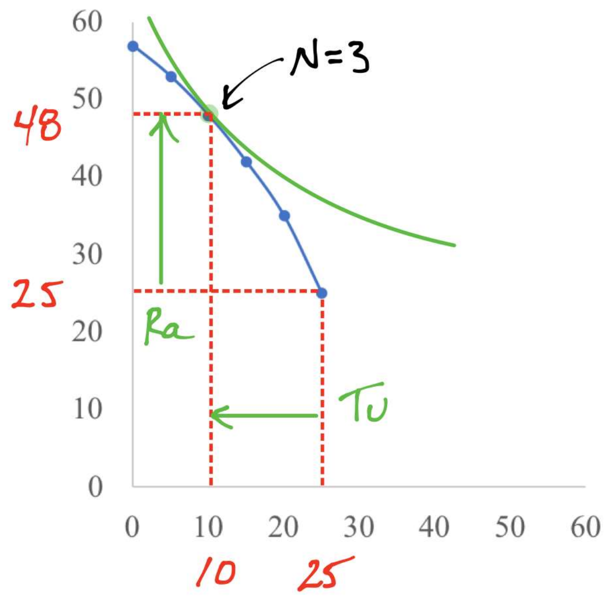
Case 3: ICs with more willingness to trade C_0 for C_1



Now, N>0

With the ICs shown:
N=3

Linking N, tuition (Tu) and the raise (Ra):



At N=3:

$Tu = 15k$
$Ra = 23k$

Key insight:

Without borrowing or saving many preferences lead to N=0

Examples: case 1 (impatient), case 2 (PC)

Now add option to borrow or save

Suppose $r = 5\%$

Now have two decisions:

1. Number of classes to take
2. Amount to borrow or save

Can think them through in that order

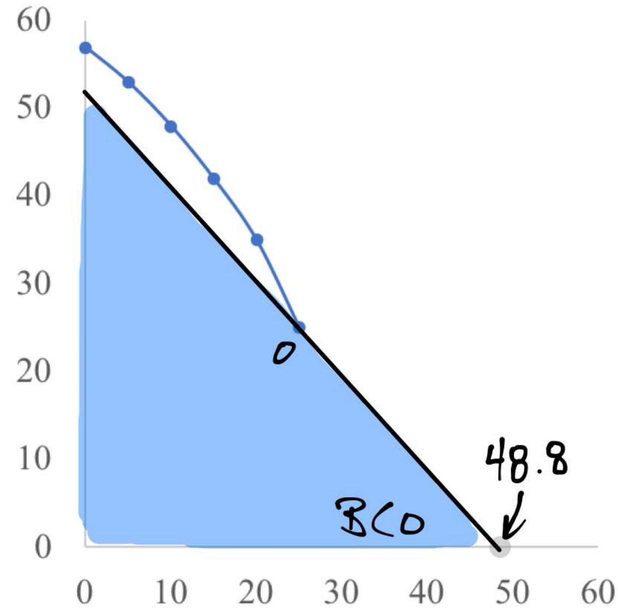
Suppose chooses N=0; what bundles are feasible?

Net income for N=0:

net

$$I_0^{net} = 25k$$
$$I_1^{net} = 25k$$

$$PVI = 25k + \frac{25k}{1.05} = 48.8k$$

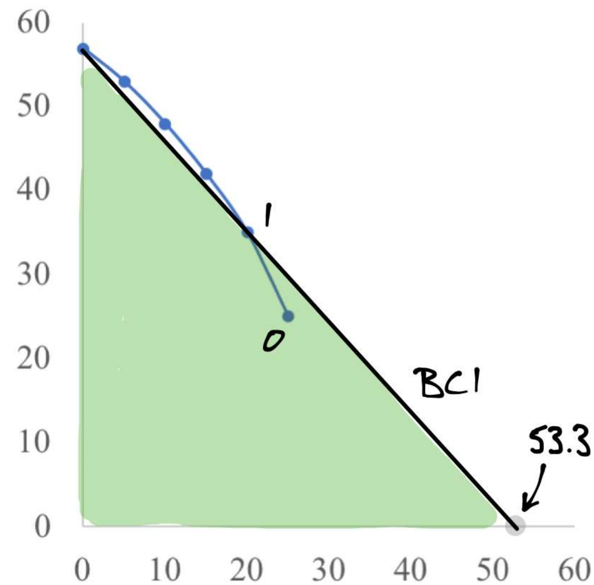


Suppose chooses N=1; what bundles are feasible?

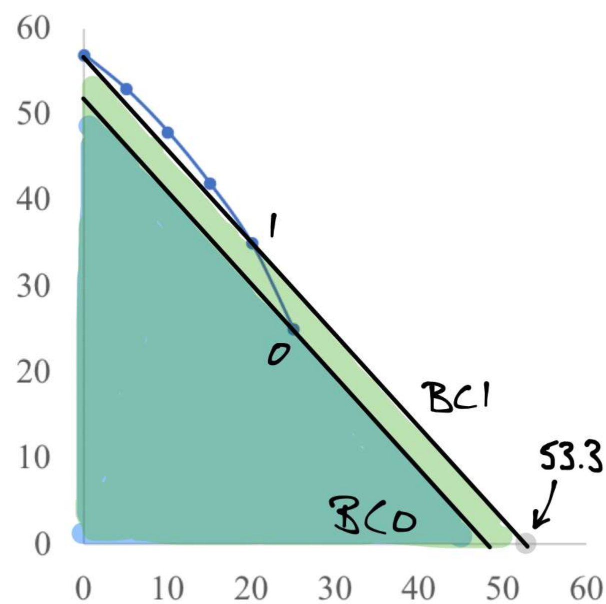
Net income for N=1:

$$I_0^{net} = 25k - 5k = 20k$$
$$I_1^{net} = 25k + 10k = 35k$$

$$PVI = 20k + \frac{35k}{1.05} = 53.3k$$



Overlaying the BC0 and BC1 sets:



Feasible set for BC1 contains:

- **All** bundles in BC0 (darker color)
- **Plus** bundles with more C_0, C_1 or both (lighter color)

Implication:

BC1 is better for *all* preferences

Technically, BC1 dominates BC0