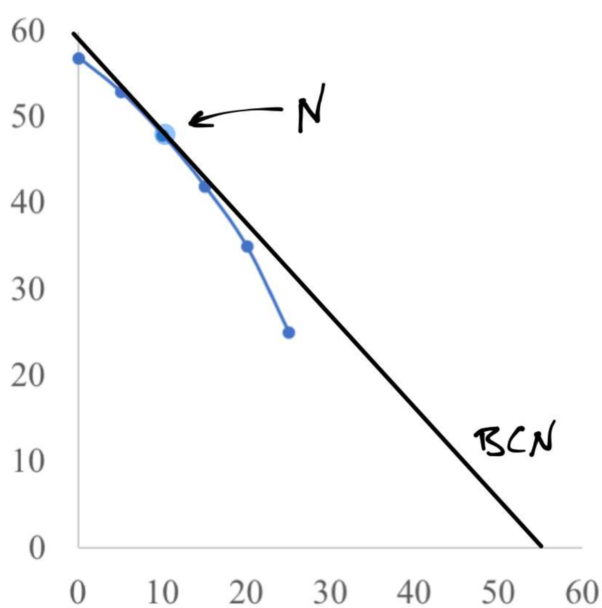
Human Capital: Finding the Best Option

Finishing the example:

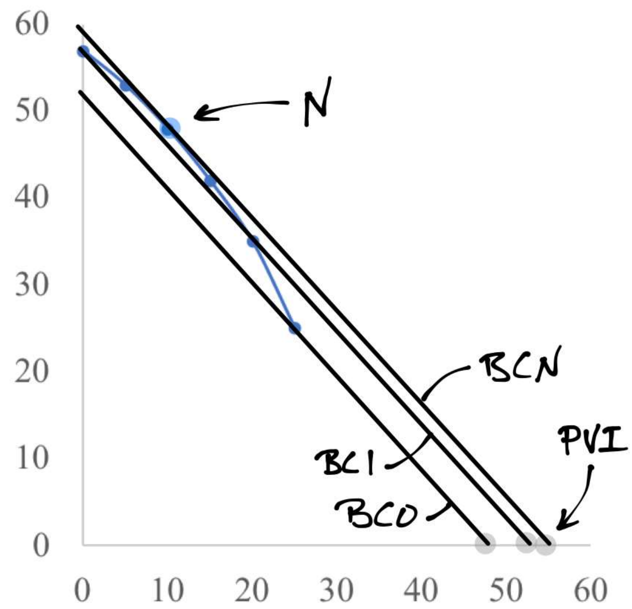
What's the best N?

The N that pushes the BC furthest out:



Finding it is surprisingly easy:

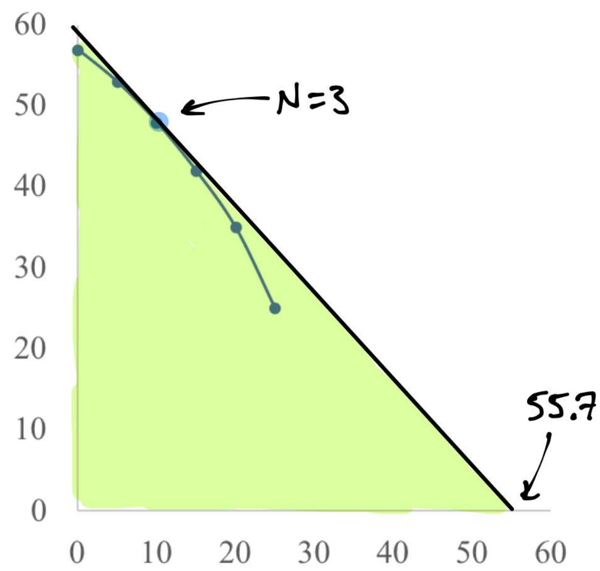
It's the net income bundle with the largest PVI:



Computing the PVIs:

Classes	I_0^{net}	I_1^{net}		PVI
0	25	25	$25 + \frac{25}{1.05}$	48.8
1	20	35	$20 + \frac{35}{1.05}$	53.3
2	15	42	$15 + \frac{42}{1.05}$	55.0
3	10	48	$10 + \frac{48}{1.05}$	55.7
4	5	53	$5 + \frac{53}{1.05}$	55.5
5	0	57	$0 + \frac{57}{1.05}$	54.3

For *all* sets of preferences, N=3 is best:



Gives the BC for choosing the *consumption* bundle C_0 and C_1:

$$C_0 + \frac{C_1}{1.05} = 55.7k$$

Human Capital: Adding Consumption Preferences

BC with N=3:

$$C_0 + \frac{C_1}{1.05} = 55.7k$$

Suppose person has PC preferences:

$$\frac{C_0}{C_1} = \frac{1}{1}$$

$$C_0 = C_1$$

Finding the optimal bundle:

$$C_0 + \frac{C_0}{1.05} = 55.7k$$

$$C_0\left(1 + \frac{1}{1.05}\right) = 55.7k$$

$$C_0 = 28.5k$$

$$C_1 = 28.5k$$

Borrowing or saving?

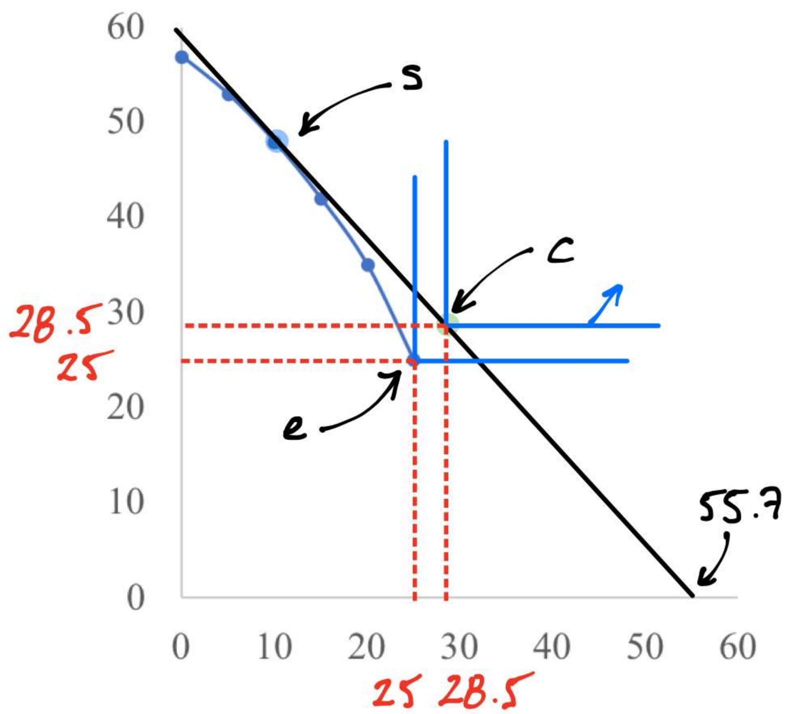
$$I_0^{net} = 10k$$

$$C_0 = 28.5k$$

Borrows in period 0:

$$B = C_0 - I_0^{net} = 28.5k - 10k = 18.5k$$

Graphing:



e = endowment
s = net income with school
c = consumption bundle

Key observation:

Chooses N=0 *without* access to borrowing
Chooses N=3 *with* access to borrowing

- Financial market separation:
  Allows human capital decision (N) to be separated from the consumption decision

Applying PV to Policies Instead of Classes

Largest PV identifies largest feasible set for policies as well

Example with three options:

Policy	Details
BAU	Provides **$100k** in 0 and 1
Option A	Costs **$10k** in 0, provides additional **$20k** in 1
Option B	Costs **$25k** in 0, provides additional **$30k** in 1

As a table of net payments:

Policy	Net in 0	Net in 1
BAU	$100k$	100k
A	$100k - 10k = 90k$	$100k + 20k = 120k$
B	$100k - 25k = 75k$	$100k + 30k = 130k$

Computing PVs at $r = 10\%$:

Policy	PV calculation	PV
BAU	$100k + \frac{100k}{1.1}$	$190.9k$
A	$90k + \frac{120k}{1.1}$	$199.1k$
B	$75k + \frac{130k}{1.1}$	$193.2k$

These are *gross* or *absolute* payoffs:

Show what actually happens under each policy, including BAU

Often convenient to measure payoff *relative to BAU*

Show *net* payoffs and compute *net* present value

Net present value (NPV):

NPVs for the example:

Policy	Difference from BAU	NPV
A	$\$199.1k - \$190.9k$	$\$8.2k$
B	$\$193.2k - \$190.9k$	$\$2.3k$

Can compute NPVs directly from changes in payoffs:

Policy	Change in 0	Change in 1	PV of changes	NPV
A	$-10k$	$+20k$	$-10k + \frac{20k}{1.1}$	$8.2k$
B	$-25k$	$+30k$	$-25k + \frac{30k}{1.1}$	$2.3k$

Approaches are equivalent:

Difference in PVs	PV of differences
$NPV = PV(A) - PV(BAU)$	$NPV = PV(A - BAU)$

BAU BAU BAU BAU

$$\left(I_0^A + \frac{I_1^A}{1+r}\right) - \left(I_0^{BAU} + \frac{I_1^{BAU}}{1+r}\right) \qquad (I_0^A - I_0^{BAU}) + \frac{(I_1^A - I_1^{BAU})}{1+r}$$

Can use whichever way is clearest and most convenient.

Bottom line:

Policy option with the highest PV (or highest NPV):

- Largest feasible set of C_0 and C_1 options
- Either or both periods can be made better off
- Pareto efficient

Extending PV to Multiple Periods

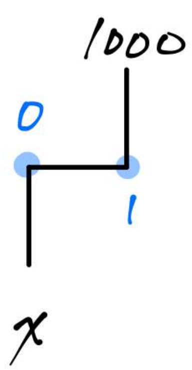
Fundamental intuition about PV:

The PV of a payment of *F* dollars at time *T*:

Amount you'd need to put in a bank at 0 in order to have *F* at *T*

Example:

Payment $1000
Period 1
$r = 10\%$

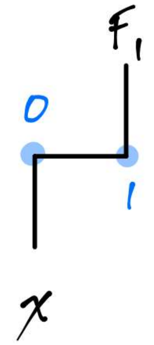


Want balance at 1 to be $1000:

$$x(1 + r) = 1000$$

$$x = \frac{1000}{1.1} = 909.09$$

Generalizing to other possible payments and interest rates:

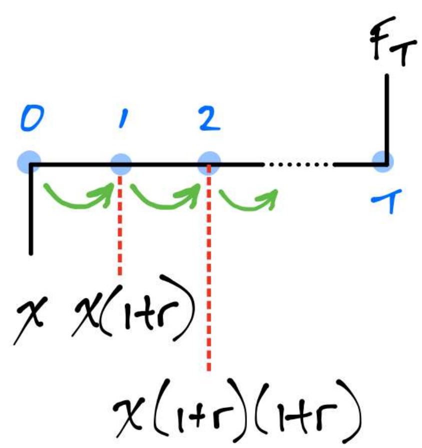


Want balance at 1 to be F_1:

$$x(1+r) = F_1$$

$$x = \frac{F_1}{1+r}$$

Generalizing to more periods:



Period	Balance
0	x
1	$x(1+r)$
2	$[x(1+r)](1+r) = x(1+r)^2$
3	$x(1+r)^3$

…	…
T	$x(1+r)^T$

Want balance at T to be F_T:

$$x(1+r)^T = F_T$$

$$x = \frac{F_T}{(1+r)^T}$$

For clarity, rename x to PV:

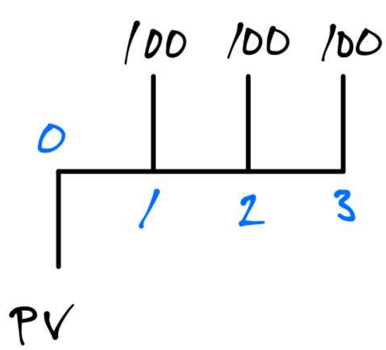
$$PV = \frac{F_T}{(1+r)^T}$$

⚠ PV formula 1

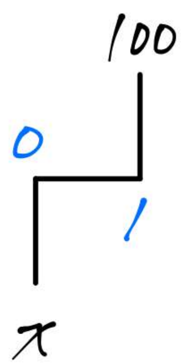
Extending to streams of multiple payments:

Example:

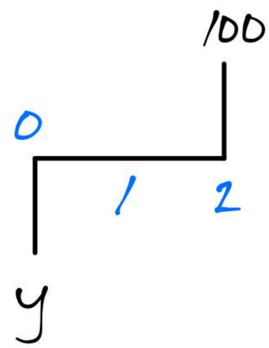
$100 in 1, 2 and 3
$r = 10\%$



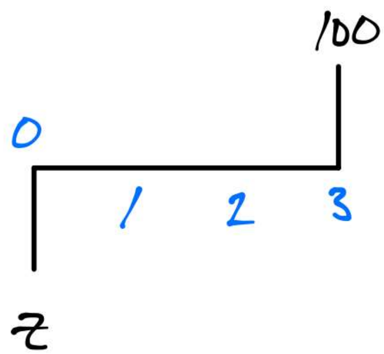
Could use 3 accounts:



$$x = \frac{100}{1.1} = 90.91$$



$$y = \frac{100}{1.1^2} = 82.64$$



$$z = \frac{100}{1.1^3} = 75.13$$

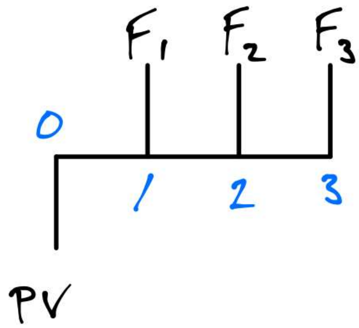
Total deposit needed:

$$PV = x + y + z$$

$$PV = \frac{100}{1.1} + \frac{100}{1.1^2} + \frac{100}{1.1^3}$$

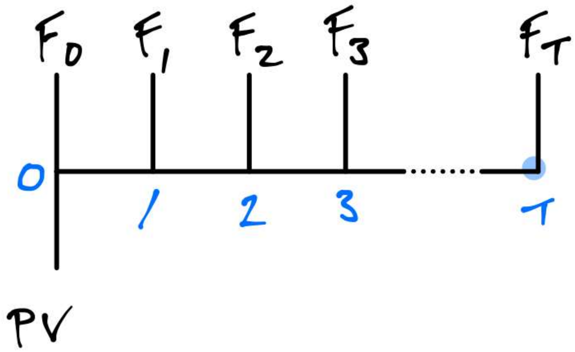
$$PV = 248.68$$

Generalizing to any three payments F_1, F_2, and F_3:



$$PV = \frac{F_1}{(1+r)^1} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3}$$

Generalizing to any finite stream with payments from 0 to T:



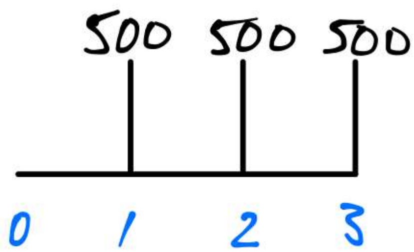
$$PV = \frac{F_0}{(1+r)^0} + \frac{F_1}{1+r} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3} + \cdots + \frac{F_T}{(1+r)^T}$$

$$PV = \sum_{t=0}^{T} \frac{F_t}{(1+r)^t}$$

⚠ PV formula 2

Example application 1: WTP for a contract

Contract delivers $500 in periods 1-3
$r = 15\%$

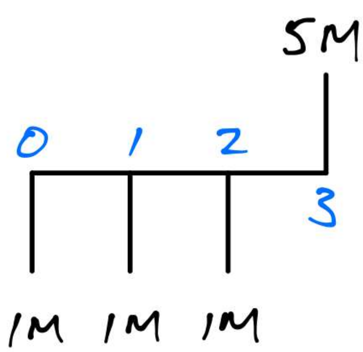


$$PV = \frac{500}{1.15} + \frac{500}{1.15^2} + \frac{500}{1.15^3} = 1142$$

Don't pay more than $1142 for contract: cheaper to get via a bank

Example application 2: PV of a construction project

Construction costs of $1M in 0, 1 and 2
Finished project worth $5M in 3
$r = 10\%$



$$PV = -1M + \frac{-1M}{1.1^1} + \frac{-1M}{1.1^2} + \frac{5M}{1.1^3}$$
$$PV = -2.736\ M + 3.757\ M$$

$$PV = 1.021\ M$$

Daily exercise on Google Classroom