

E: PV refresher 1

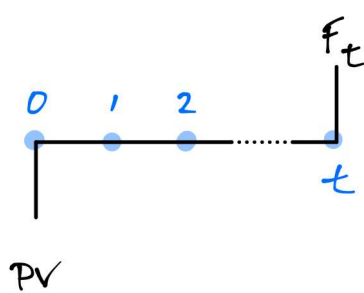
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Interpretation of PV:

The amount of money needed in a bank account today (time 0) in order to deliver a target sequence of payments in the future.

PDF quick reference guide posted under class materials

Formula 1: Single payment:



$$PV = \frac{F_t}{(1 + r)^t}$$

Example 1: PV of a single payment:

F \$10k
t Year 5
r 10%

$$PV = \$10,000 / (1.1)^5 = \$6,209$$

Example 2: using PV as a benchmark for evaluating policies:

Proposed policy:

Cost \$3000 today
Delivers \$5000 in year 4

Cost of alternative using a bank account at $r=10\%$?

$$PV = \$5000 / (1.1)^4 = \$3415$$

Conclusion:

Project is \$415 cheaper than the bank.

Expressing via net present value (NPV):

$$NPV = PV \text{ of benefits} - PV \text{ of costs}$$

$$NPV = \$3415 - \$3000 = \$415$$

Project produces a net gain of \$415

Example 3: arbitrage trading:

Suppose know price of oil is rising:

$$P_0 = \$50$$

$$P_1 = \$60$$

$$r = 10\%$$

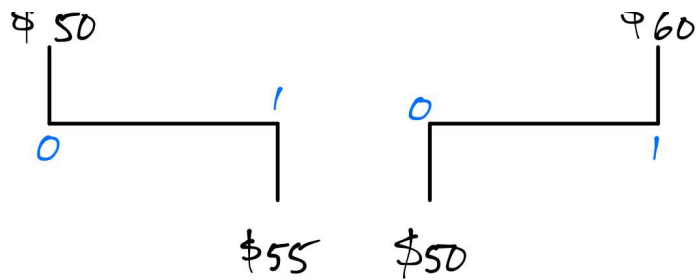
Arbitrage trade:

1. Borrow \$50

2. Buy oil at 0

Loan Oil

3. Sell at 1
4. Repay loan



$$NPV = \$60/1.1 - \$55/1.1 = \$4.55$$

Profitable: returns exceed interest cost

Formula 2: extension to streams with multiple payments from 0 to T :

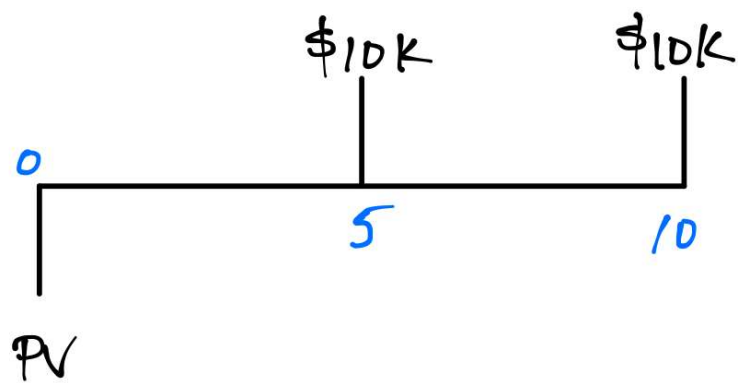
The diagram shows a horizontal line representing time. At time 0, there is a vertical line with a blue '0' below it and a '\$F_0\$' above it. At time 1, there is a vertical line with a blue '1' below it and a '\$F_1\$' above it. At time 2, there is a vertical line with a blue '2' below it and a '\$F_2\$' above it. At time 3, there is a vertical line with a blue '3' below it and a '\$F_3\$' above it. A dotted line follows, and at time T, there is a vertical line with a blue 'T' below it and a '\$F_T\$' above it. Below the diagram, the text 'PV' is written.

$$PV = \sum_{t=0}^T \frac{F_t}{(1+r)^t}$$

PV of the stream is the sum of the individual PVs

Example 4: two payments

Payments each \$10,000
 One in year 5, one in year 10
 $r = 5\%$



$$PV = \$10,000/1.05^5 + \$10,000/1.05^{10} = \$13,974$$