C: Solving equations

Very frequent task in numerical analysis:

Given a function $f(x)$
Find an x^* such that $f(x^*) = 0$

Some examples:

- IRR calculation:
  Find an r^* such that $NPV(r^*) = 0$

- Market equilibrium:
  Find a price p^* such that $Q^D(p^*) - Q^S(p^*) = 0$

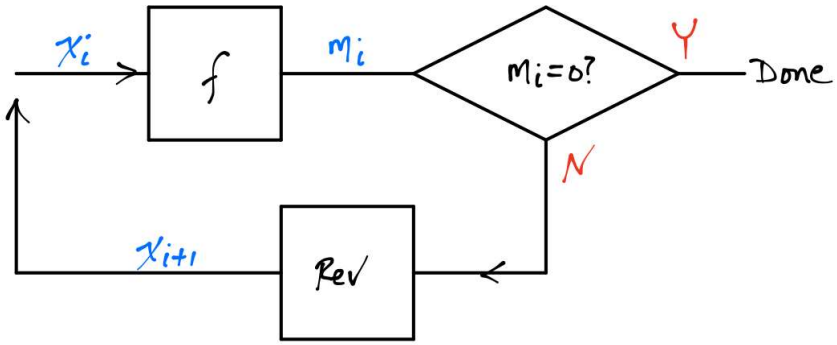
- Carbon tax t^* to reduce emissions to a target level
- Clean energy credit c^* that causes renewable electricity to hit 50%
- Low income tax credit z^* to eliminate regressivity of a tax
- … many, many more

Core technique is Newton's Method:

Iterative process:

1. At trial i guess x_i
2. Evaluate f at the guess to find "miss distance" m_i: $f(x_i) = m_i$
3. If m_i isn't 0, revise the guess to x_{i+1} and try again

Schematically:

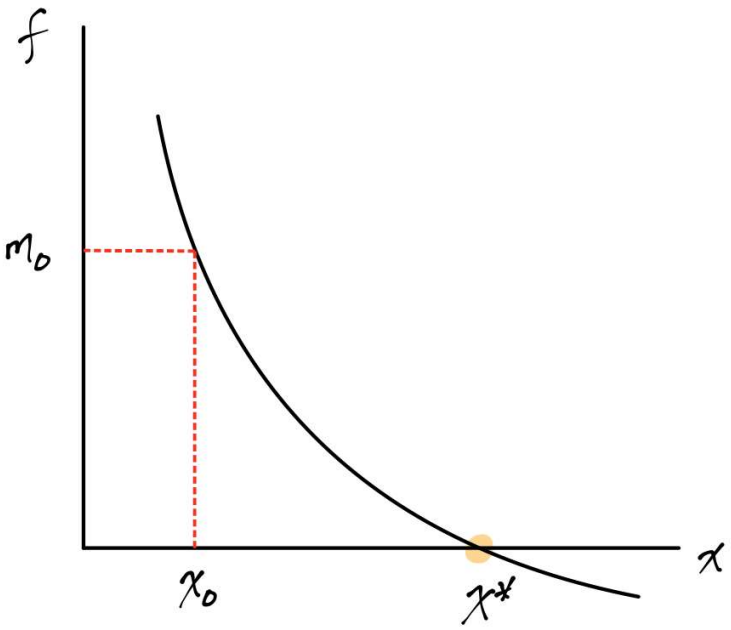


Heart of Newton's Method is the revision procedure:

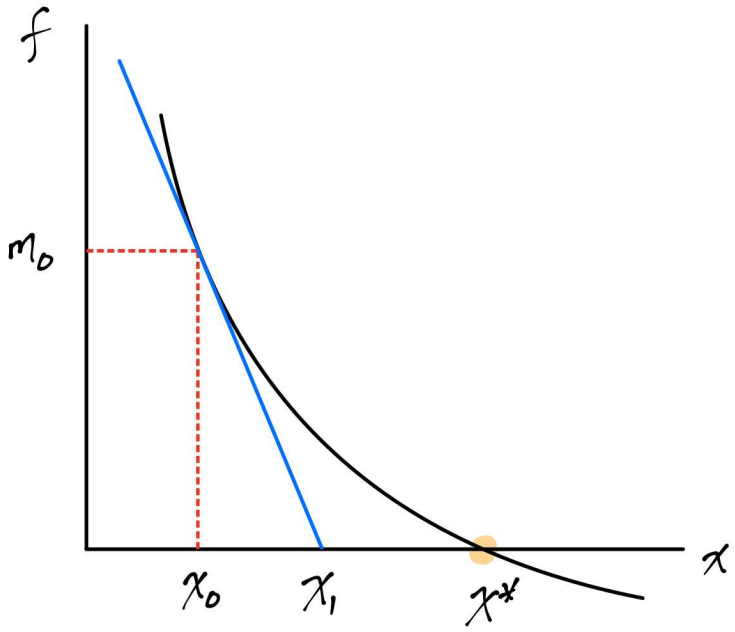
- Uses the slope of the function at x_i
- Very fast in a wide range of cases

Graphically:

Initial guess and miss distance:



Using the slope of the function to find a new guess:



Why try this?

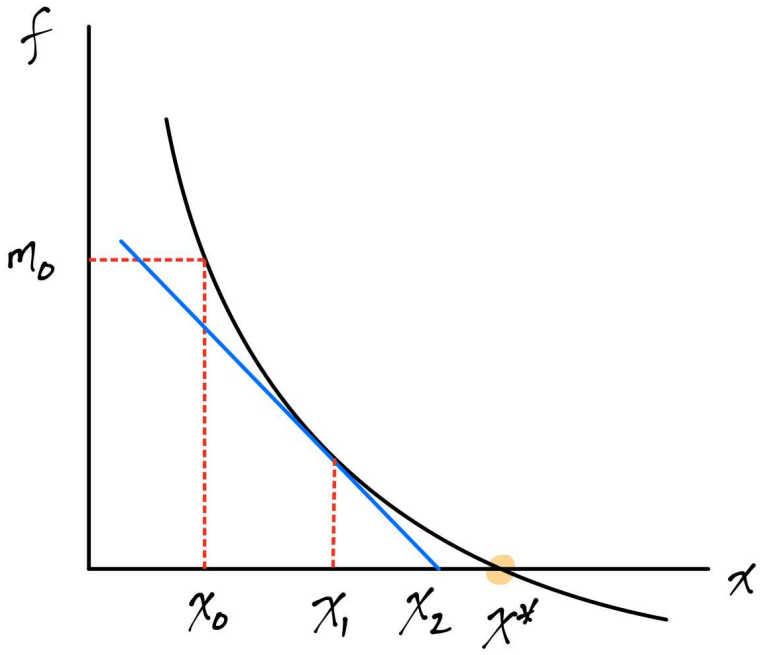
Hits x^* on the first try if f is linear

Still makes a big improvement for many other functions.

Algebraically, if the slope at x_0 is s_0, can show:

$$x_1 = x_0 - \frac{m_0}{s_0}$$

Updating x, m and s and generating a new guess:



Next guess:

$$x_2 = x_1 - \frac{m_1}{s_1}$$

Repeat until $f(x_i) = 0$

Outline of invoking Newton in Python:

1. Import the scipy module
2. Define a miss distance function
3. Create a starting guess
4. Call the scipy Newton's Method function; will return x^*

In code:

```
1  import scipy.optimize as opt

2  def miss_func( guess, … ):
       …
       return miss

3  start_guess = …

4  sol = opt.newton( miss_func, start_guess, … )
```

Variable sol will be the solution x^*

Minor complication: passing additional variables to the miss function

The miss function usually takes *several arguments*:

Example: npv(r, cashflow)

opt.newton() iterates over guesses of the **first argument**
Example: r

To pass **additional arguments** give them as a list via optional parameter args:
opt.newton(…., args=[cashflow])

Additional arguments are not altered by opt.newton()