E: Imperfect information

Extend analysis to handle information that may be incorrect

Key intuition:

Information reduces but does not eliminate uncertainty

Example 1: testing a used car

Car price is $2500
Car condition is uncertain:

State	Probability	Value
Good (G)	30%	$4000
Bad (B)	70%	$1000

Test is available:

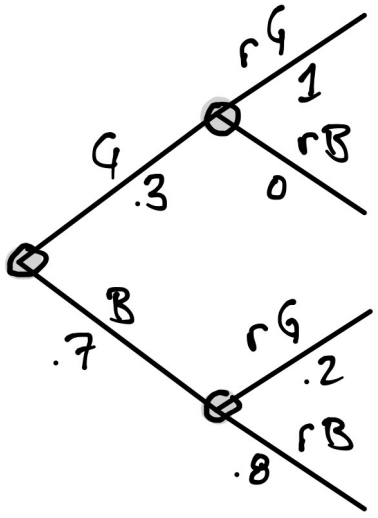
Costs $100
Reports whether the car is good (rG) or bad (rB)

Test characteristics:

- Never reports problems if the car is good
  Never says rB if car is G
  No false positives

- Misses problems in bad cars 20% of the time
  May report rG when car is B
  Some false negatives

Report depends on true condition so it's to the right in the tree:

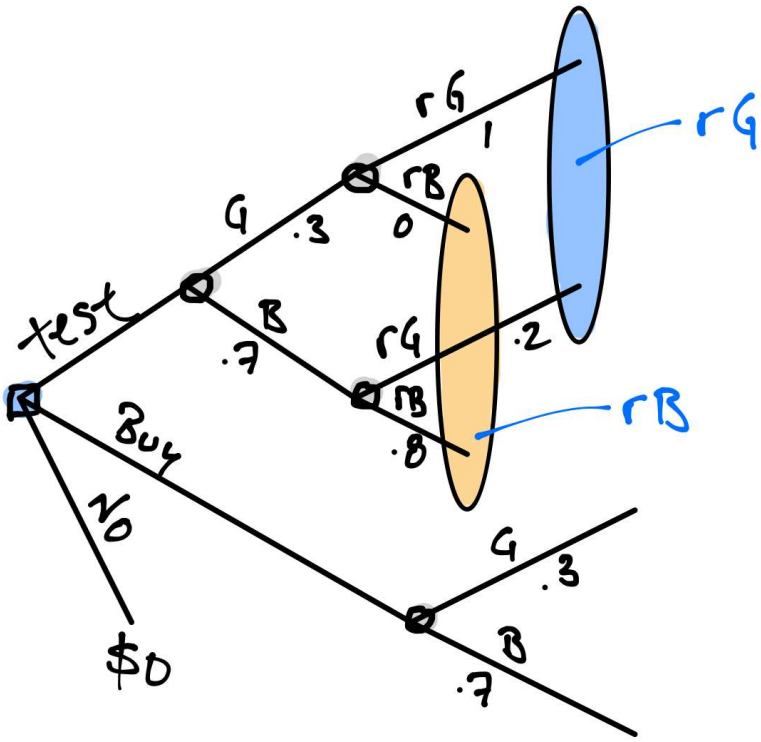


Expressing as a table:

True state	Report	Prob of report
G	rG rB	100% 0%
B	rG rB	20% 80%

Start building the decision tree:

Two *information sets*:



Blue:
have report rG

Orange:
have report rB

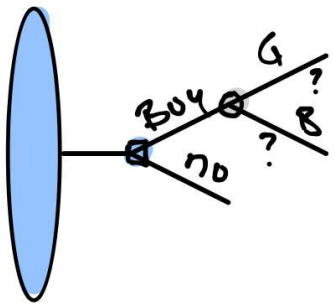
After test results received:

Report rG: Uncertain: car *might* be good

Report rB: Certain: car is *definitely* bad

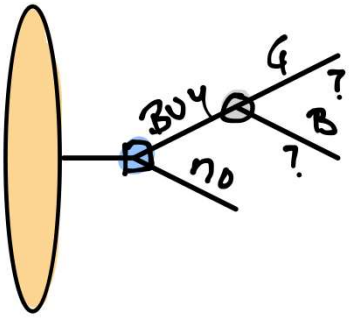
Either case, next node is decision whether to buy:

rG:



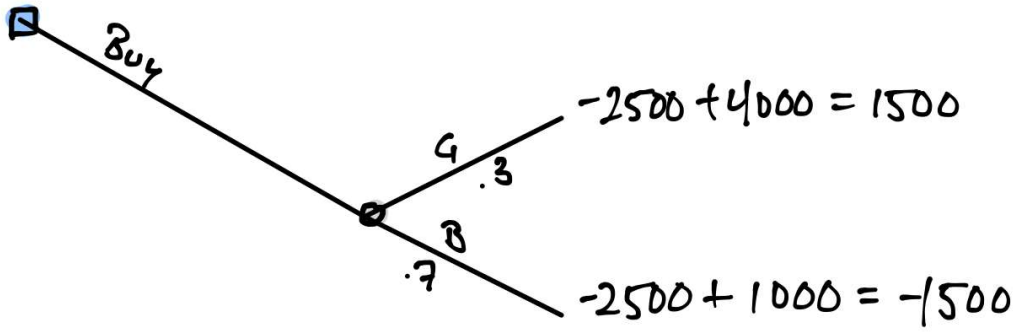
Probabilities:
will need to
calculate

rB:



Probabilities:
pr G = 0
pr B = 100%

Evaluate Buy branch to simplify tree:

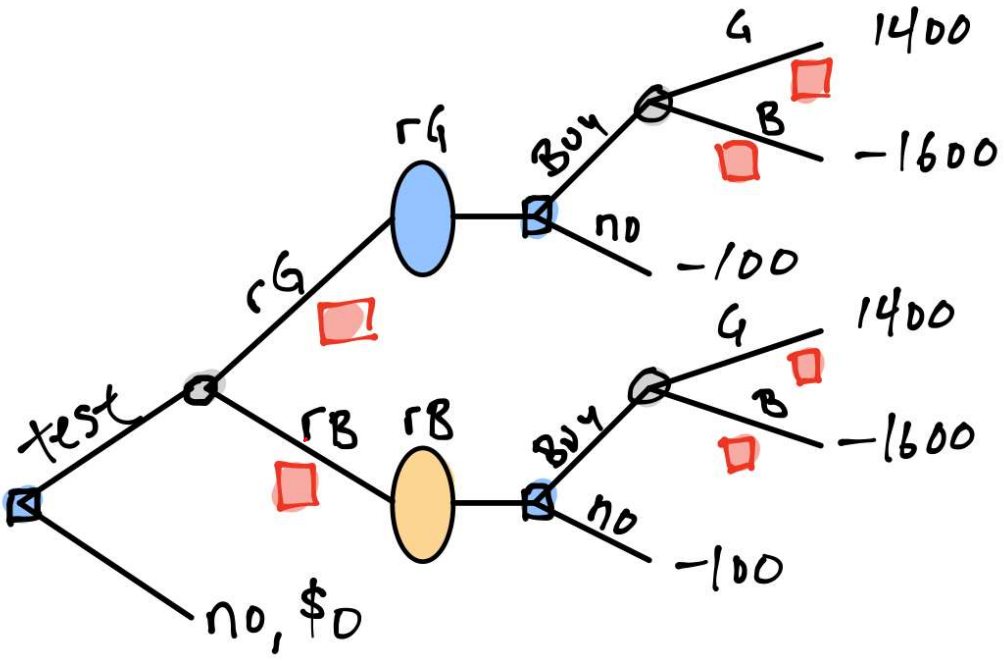


EV = 0.3*1500 + 0.7*(-1500) = -600

Don't buy without the test

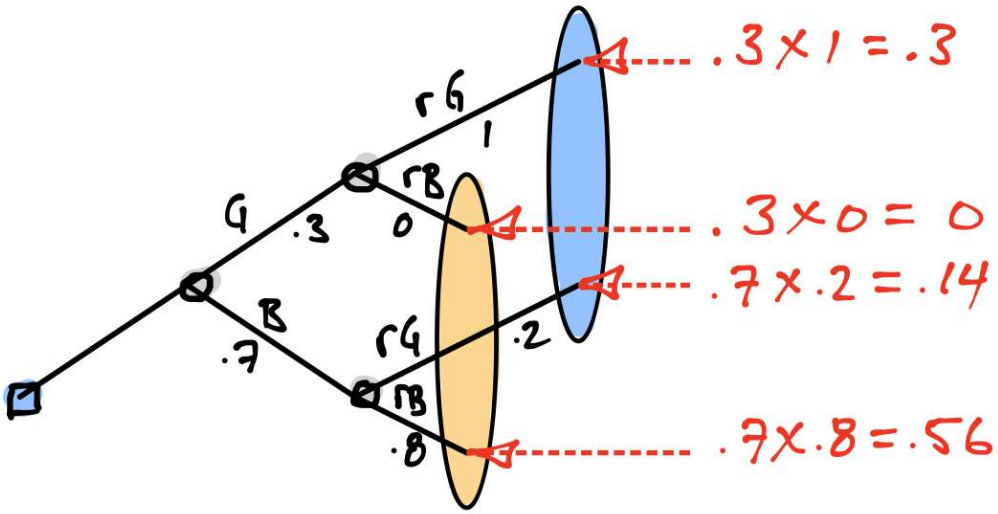
Can redraw tree from buyer's perspective:

Payoff if *test, buy, car is good*: - 100 - 2500 + 4000 = 1400
Payoff if *test, buy, car is bad*: -100 - 2500 + 1000 = -1600



Red boxes: probabilities to be determined

Step 1: probabilities of individual information set endpoints:



Checking the sum: 0.3 + 0 + 0.14 + 0.56 = 1

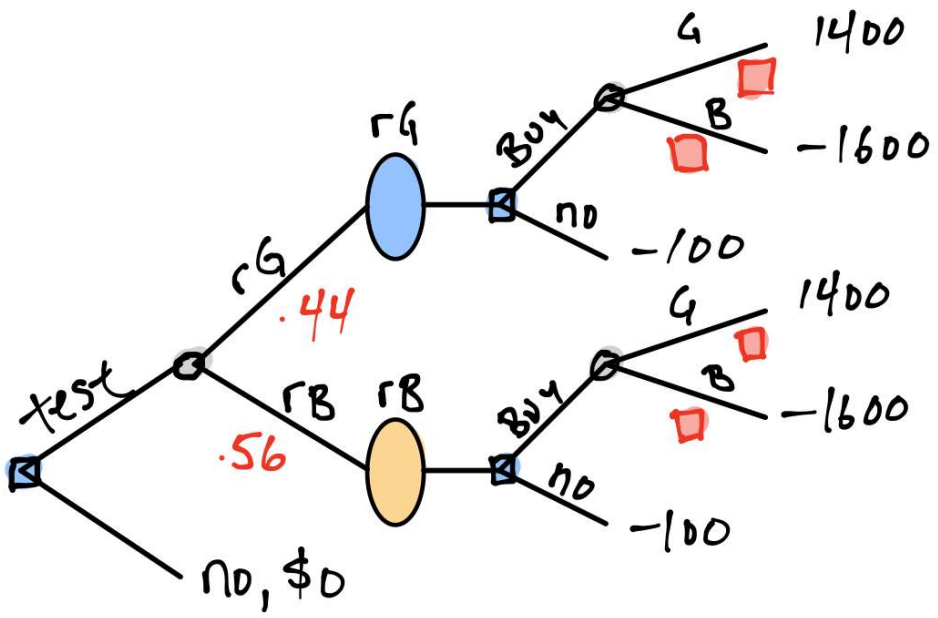
Step 2: probabilities of information sets rG, rB:

probability of rG: 0.3 + 0.14 = 0.44

probability of rB: 0 + 0.56 = 0.56

Checking the sum: 0.44 + 0.56 = 1

Adding to the tree:



Step 3: calculate conditional probabilities after rG and rB:

Conditional probability:

Probability of **true state** given **reported state**

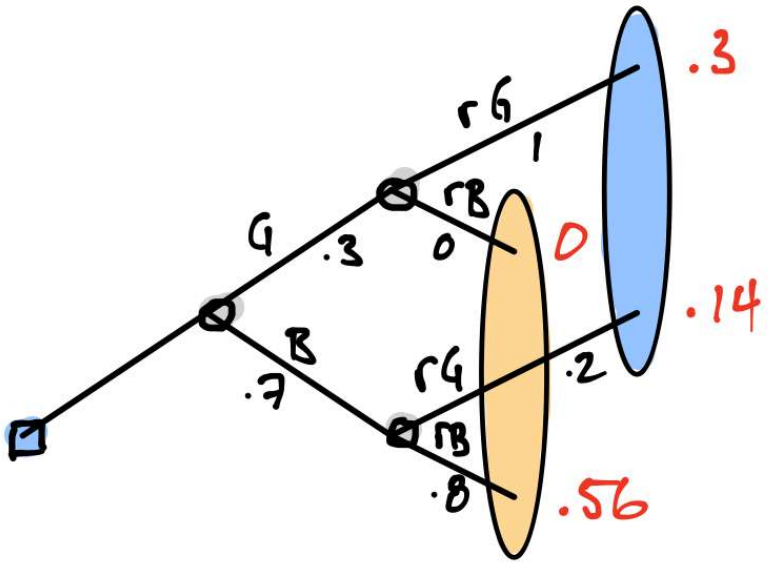
Example:

Probability car is actually **good** (G) when **reported good** (rG)?

Formally an application of Bayes Rule

Will be the share of G cars in those with rG reports

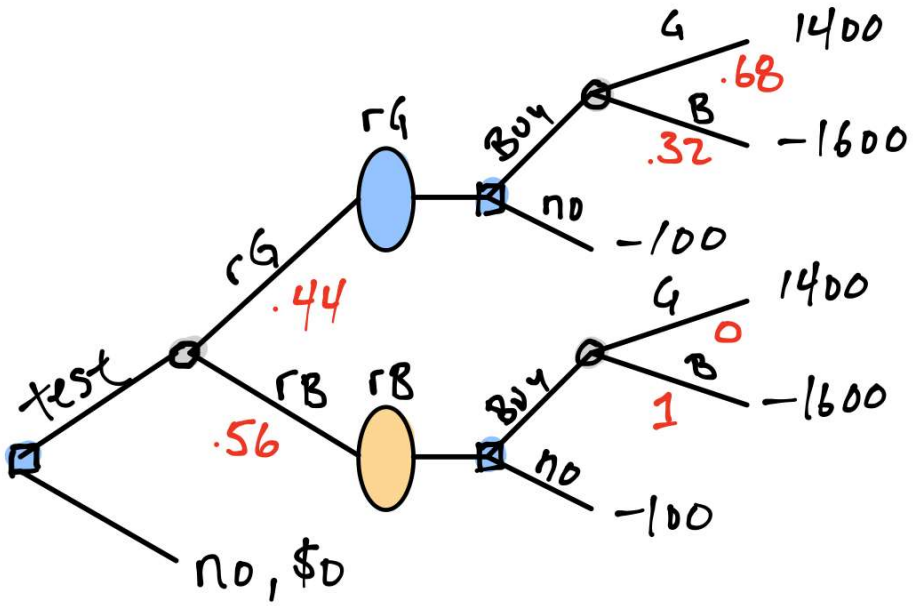
Find from original tree:



Prob G if rG:
0.3/0.44 = 0.68
Prob B if rG:
0.14/0.44 = 0.32

Prob G if rB:
0/0.56 = 0
Prob B if rB:
0.56/0.56 = 1

Adding to the buyer's view:



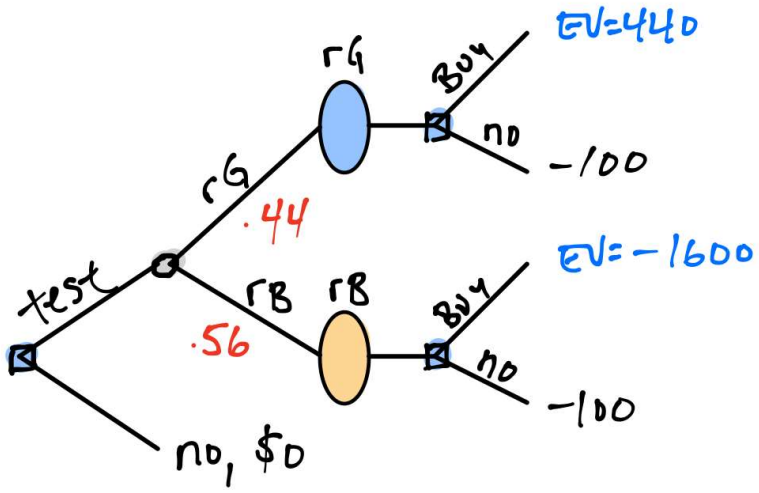
Step 4: evaluate finished tree:

Computing EVs at right:

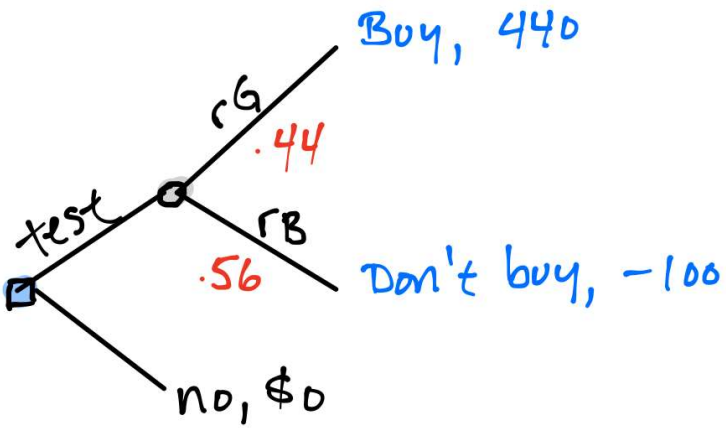
Buy if rG: 0.68*1400 + 0.32*(-1600) = 440

Buy if rB: 0*1400 + 1*(-1600) = -1600

Updating the tree:

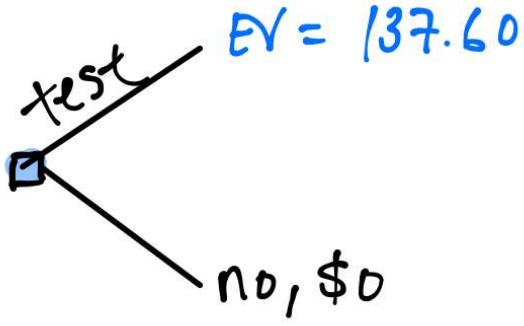


Evaluating right-most choice nodes:



Evaluating again:

EV = 0.44*440 + 0.56*(-100) = 137.6



Conclusion:

Buy the test