E: Conditional probabilities

Application: the Monty Hall problem

From a long-running game show: *Let's Make a Deal*
Possibly best illustration ever of conditional probabilities

Situation:

- Three closed doors with a new car behind one
- Contestant chooses a door
- Monty opens one of the others: no prize
- Asks the contestant if they'd like to switch

Intuitive reaction:

- No reason to change: the prize hasn't moved
- Must now have 50/50 chance

Flat wrong!

- Overlooks the fact that Monty's choice is **not random**
- **Never** opens the door with the prize
- As a result, choice reveals information

Analysis:

Notation:

Contestant's initial actions:

C1, C2, C3 = choose door 1, 2 or 3

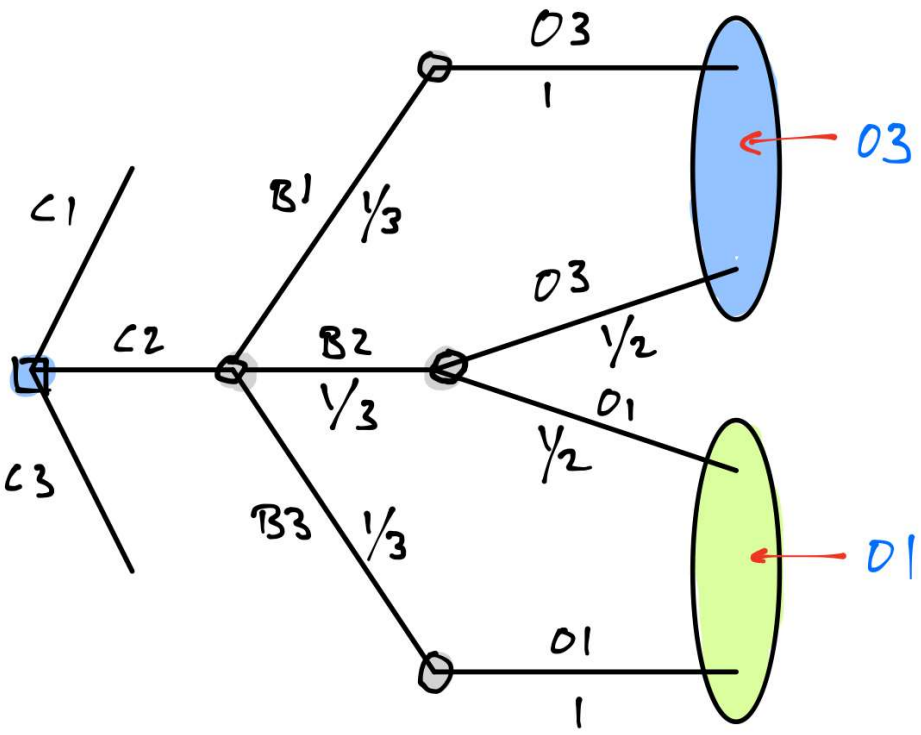
State: true location of the prize:

B1, B2, B3 = behind door 1, 2 or 3

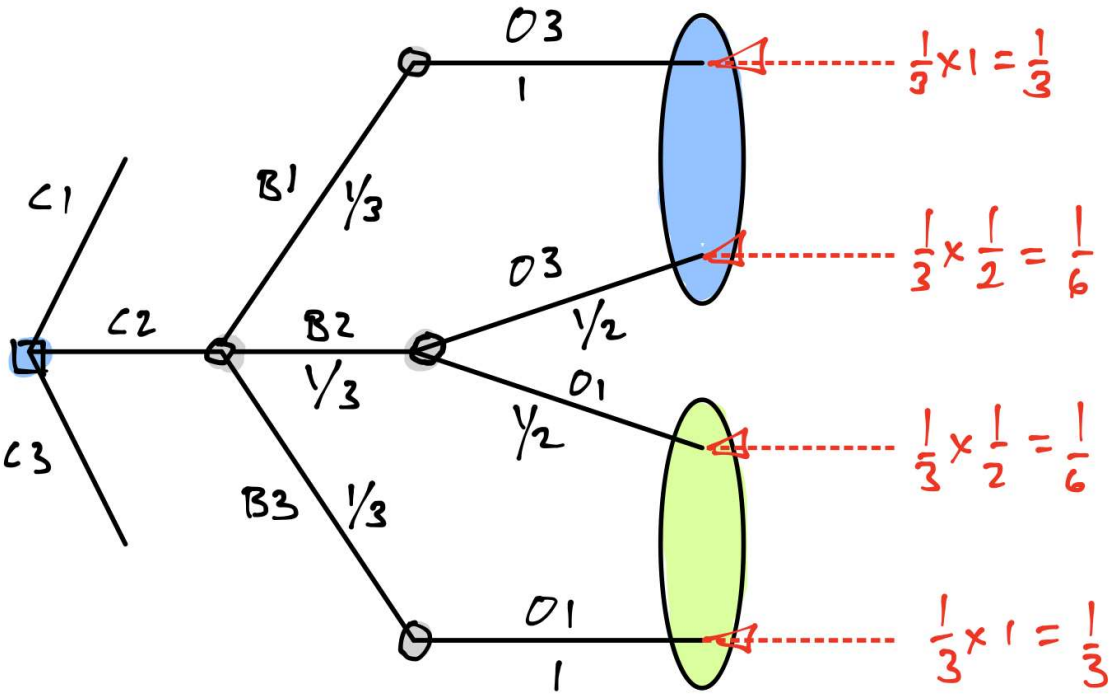
Monty's actions:

O1, O2, O3 = open door 1, 2 or 3

Tree if contestant chooses 2:



Computing the unconditional probabilities:



Computing the probabilities of O3 and O1:

Prob of O3 = 1/3 + 1/6 = 0.5
Prob of O1 = 1/6 + 1/3 = 0.5

Computing the conditional probabilities of B2 and B1 for O3:

Hang onto door 2?

Prob B2 given O3: $\frac{1/6}{1/2} = 1/3$

Switch to door 1?

Prob B1 given O3: $\frac{1/3}{1/2} = 2/3$

Conclusion:

Dramatically better to switch: probability is **double**

Monty's choice reveals very valuable information

Exercise on GC