Applying PV to Policies Instead of Classes

Largest PV identifies largest feasible set for policies as well

Example with three options:

Policy	Details
BAU	Provides **$100k** in 0 and **$100k** in 1
Option A	Costs **$10k** in 0 relative to BAU, provides additional **$20k** in 1
Option B	Costs **$25k** in 0 relative to BAU, provides additional **$30k** in 1

As a table of net payments:

Policy	Net in 0	Net in 1
BAU	$100k$	100k
A	$100k - 10k = 90k$	$100k + 20k = 120k$
B	$100k - 25k = 75k$	$100k + 30k = 130k$

Computing PVs at $r = 10\%$:

Policy	PV calculation	PV	Ranking
BAU	$100k + \frac{100k}{1.1}$	$190.9k$	
A	$90k + \frac{120k}{1.1}$	$\mathbf{199.1k}$	Best
B	$75k + \frac{130k}{1.1}$	$193.2k$	Better

These are *gross* or *absolute* payoffs:

- Show what *actually happens* under each policy, including BAU

Often convenient to measure payoff *relative to BAU*

- Show *changes from BAU as **net** payoffs*
- Use to compute **net present value (NPV)**

Net present value:

NPVs for the example:

Policy	Policy PV	BAU PV	NPV	Ranking
BAU	$190.9k$	$190.9k$	0	
A	$199.1k$	$190.9k$	$\mathbf{8.2k}$	Best
B	$193.2k$	$190.9k$	$2.3k$	Better

Can compute NPVs directly from changes in payoffs:

Policy	Change in 0	Change in 1	PV of changes	NPV
A	$-10k$	$+20k$	$-10k + \frac{20k}{1.1}$	$\mathbf{8.2k}$
B	$-25k$	$+30k$	$-25k + \frac{30k}{1.1}$	$2.3k$

Approaches are always equivalent:

Difference in PVs	PV of differences

$$NPV = PV(A) - PV(BAU) \qquad NPV = PV(A - BAU)$$

$$\left(I_0^A + \frac{I_1^A}{1+r}\right) - \left(I_0^{BAU} + \frac{I_1^{BAU}}{1+r}\right) \qquad (I_0^A - I_0^{BAU}) + \frac{(I_1^A - I_1^{BAU})}{1+r}$$

Can use whichever way is clearest and most convenient.

Bottom line:

Policy option with the highest PV or highest NPV:

- Largest feasible set of C_0 and C_1 options
- Either or both periods can be made better off
- *Pareto efficient*

Extending PV to Multiple Periods

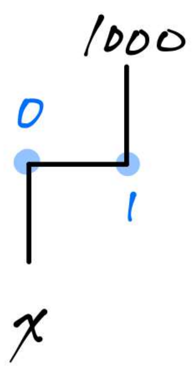
Fundamental intuition about PV:

PV of payment *F* at time *T*:

Size of bank deposit *needed at 0* to *have F at T*

Example:

Payment \$1000
Period 1
$r = 10\%$

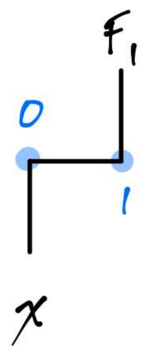


Want balance at 1 to be \$1000:

$$x(1 + r) = 1000$$

$$x = \frac{1000}{1.1} = 909.09$$

Generalizing to other possible payments and interest rates:

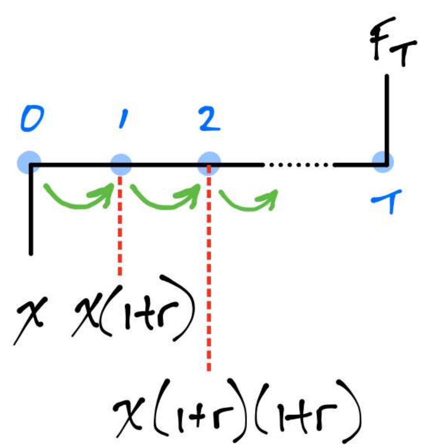


Want balance at 1 to be F_1:

$x(1+r) = F_1$

$x = \dfrac{F_1}{1+r}$

Generalizing to more periods:



Period	Balance
0	x
1	$\boldsymbol{x(1+r)}$
2	$[\,\boldsymbol{x(1+r)}\,](1+r) = x(1+r)^2$
3	$x(1+r)^3$

…	…
T	$x(1+r)^T$

Want balance at T to be F_T:

$$x(1+r)^T = F_T$$

$$x = \frac{F_T}{(1+r)^T}$$

For clarity, rename x to PV:

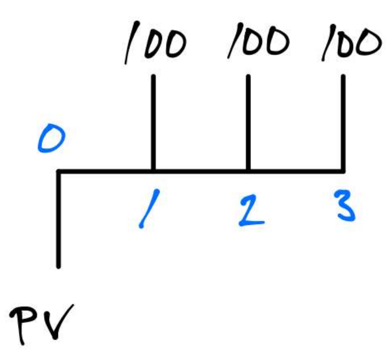
$$\boxed{PV = \frac{F_T}{(1+r)^T}}$$

⚠ PV formula 1

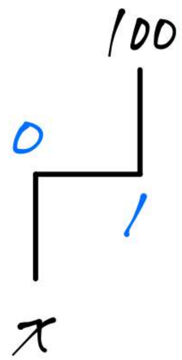
Extending to streams of multiple payments:

Example:

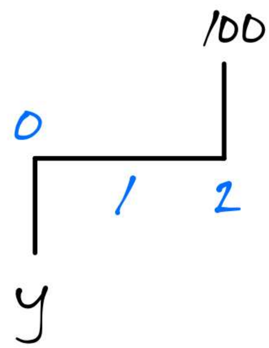
Want \$100 in 1, 2 and 3
$r = 10\%$



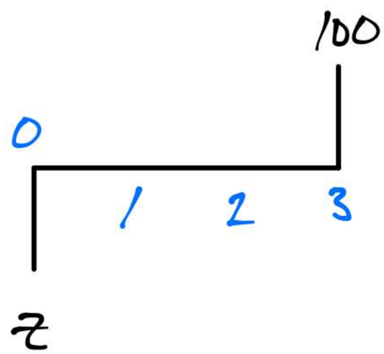
Could use 3 accounts:



$$x = \frac{100}{1.1} = 90.91$$



$$y = \frac{100}{1.1^2} = 82.64$$



$$z = \frac{100}{1.1^3} = 75.13$$

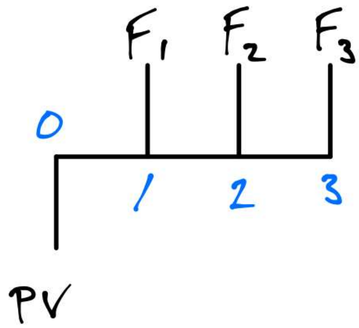
Total deposit needed:

$$PV = x + y + z$$

$$PV = \frac{100}{1.1} + \frac{100}{1.1^2} + \frac{100}{1.1^3}$$

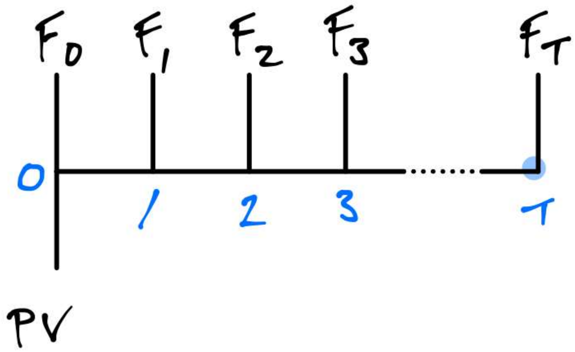
$$PV = 248.68$$

Generalizing to any three payments F_1, F_2, and F_3:



$$PV = \frac{F_1}{(1+r)^1} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3}$$

Generalizing to any finite stream with payments from 0 to T:



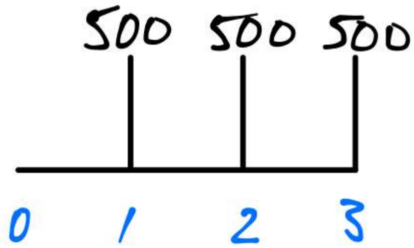
$$PV = \frac{F_0}{(1+r)^0} + \frac{F_1}{1+r} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3} + \cdots + \frac{F_T}{(1+r)^T}$$

$$\boxed{PV = \sum_{t=0}^{T} \frac{F_t}{(1+r)^t}}$$

⚠ PV formula 2

Example application 1: WTP for a contract

Contract delivers $500 in periods 1-3
$r = 15\%$

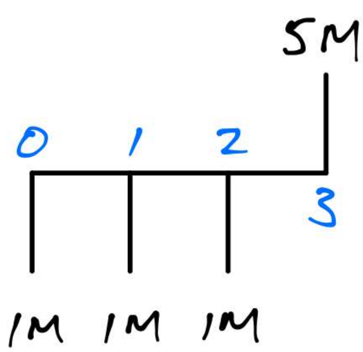


$$PV = \frac{500}{1.15} + \frac{500}{1.15^2} + \frac{500}{1.15^3} = 1142$$

Don't pay more than $1142 for contract: cheaper to get via a bank

Example application 2: PV of a construction project

Construction costs of $1M in 0, 1 and 2
Finished project worth $5M in 3
$r = 10\%$



$$PV = -1M + \frac{-1M}{1.1^1} + \frac{-1M}{1.1^2} + \frac{5M}{1.1^3}$$

$$PV = -2.736\ M + 3.757\ M$$

$$PV = 1.021\ M$$

Daily exercise on Google Classroom