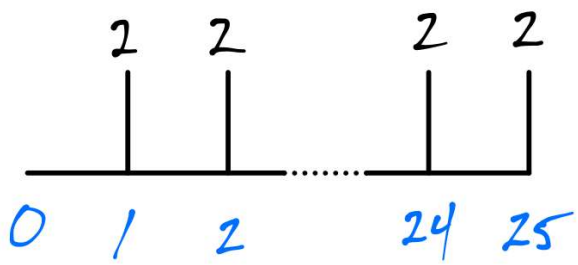
Formula 5: Long but Finite Stream

Special stream 3:

Identical payments running **from 1 to T**

Example:

Policy produces **$2M/year** in **1-25**
$r = 5\%$



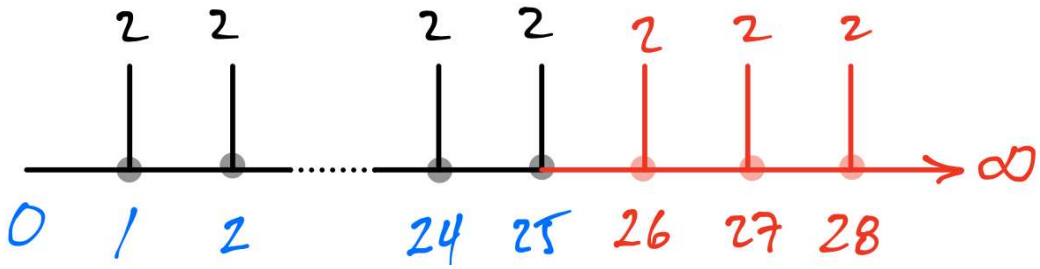
Can find the PV using formulas 3 and 4

1. Compute PV of stream *as if it were infinite* (formula 3)
2. *Subtract* the PV of *payments after year 25* (formula 4)

Step 1:

Call original stream "A" and extend it to create an infinite stream, "B":

B:



PV of B is easy using formula 3:

$$PV_B = \frac{F}{r}$$

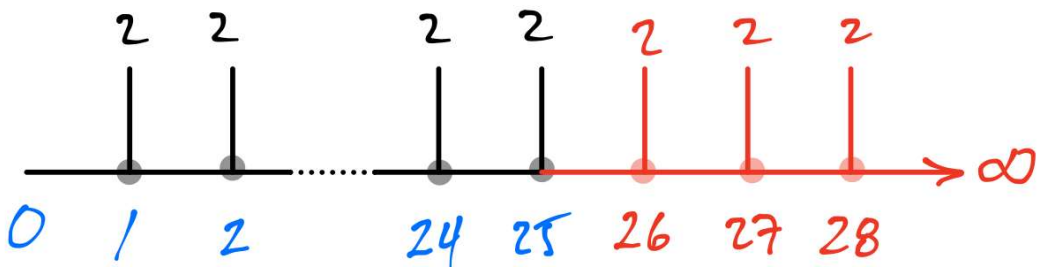
$$PV_B = \frac{2\ M}{0.05} = 40\ M$$

Larger than true value of A since extra payments are included

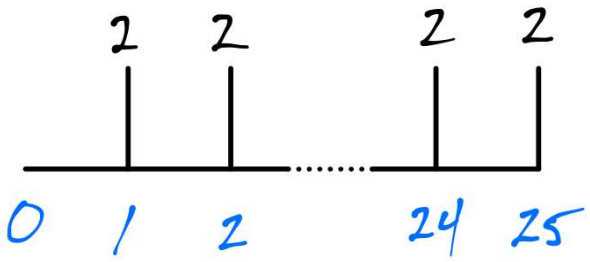
Step 2:

Stream B is composed of two pieces, A and the extra payments C:

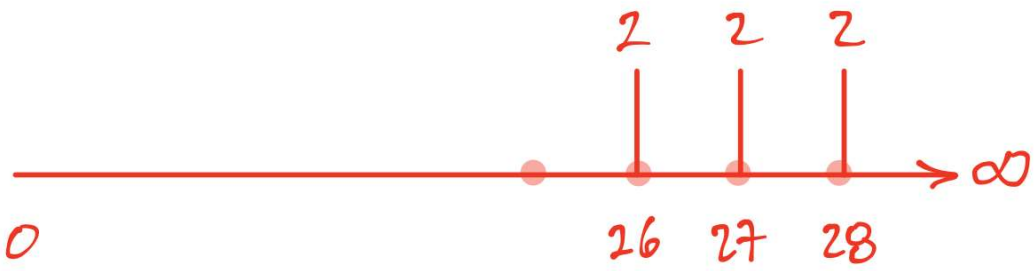
B:



A:



C:



Can calculate PV_C using formula 4 with $T = 25$:

$$PV_C = \frac{\frac{F}{r}}{(1+r)^T}$$

$$PV_C = \frac{\frac{2}{0.05}}{1.05^{25}} = 11.8\ M$$

Know the PVs of the three streams are linked:

$$PV_B = PV_A + PV_C$$

Solve for PV_A:

$$PV_A = PV_B - PV_C$$

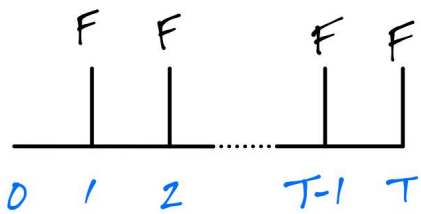
$$PV_A = 40\ M\ - 11.8\ M = 28.2\ M$$

Nice interpretation:

Target PV is the infinite PV corrected for missing red payments

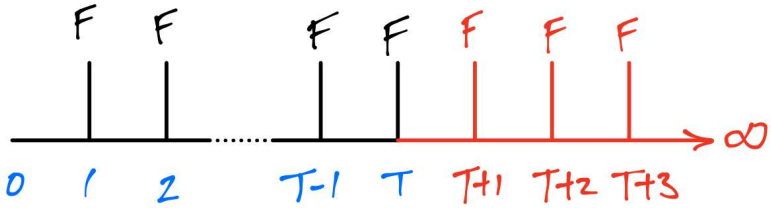
Generalizing:

A:



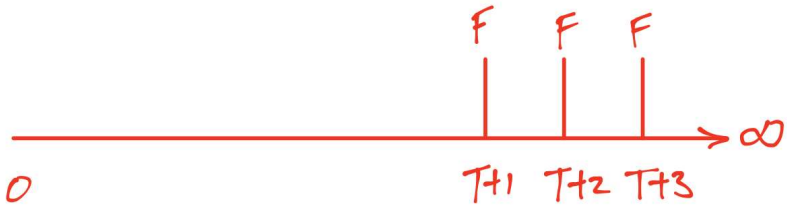
$$PV_A$$

B:



$$PV_B = \frac{F}{r}$$

C:



$$PV_C = \frac{\frac{F}{r}}{(1+r)^T}$$

$$PV_A = PV_B - PV_C$$

$$PV_A = \frac{F}{r} - \frac{\frac{F}{r}}{(1+r)^T}$$

$$PV_A = \frac{F}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

⚠ PV formula 5

Example:

A bridge provides **$10 M/year** of benefits for 30 years in **years 1-30**
$r = 5\%$

Using the two-step calculation:

$$PV = \frac{F}{r} - \frac{\frac{F}{r}}{(1+r)^T}$$

$$\frac{F}{r} = \frac{10\ M}{0.05} = 200\ M$$

$$PV = 200\ M - \frac{200\ M}{1.05^{30}} = 200\ M\ - 46.3\ M = 153.7\ M$$

Single step calculation:

$$PV = \frac{10\ M}{0.05}\left(1 - \frac{1}{1.05^{30}}\right)$$

$$PV = 200\ M * (1 - 0.231) = 200\ M * 0.769 = 153.7\ M$$

Quick Reference Guide on the PV resources page:
https://wilcoxen.maxwell.insightworks.com/pages/4350.html

Daily exercise on Google Classroom