Choice Under Uncertainty

Arises in many contexts:

Long term policies or infrastructure (e.g., highway):
Future conditions are inherently uncertain

New policies (e.g., mask mandate):
May not know effectiveness
May not know costs or benefits accurately

Heterogeneous agents (e.g., insurance):
Agent type may be unknown

To handle, use two key tools:

1. Decision trees:
   Analytical diagrams of actions, uncertainties, and payoffs

2. Expected value:
   Used when analyzing uncertain branches of decision trees

Example 1: buying a used car

Car characteristics:

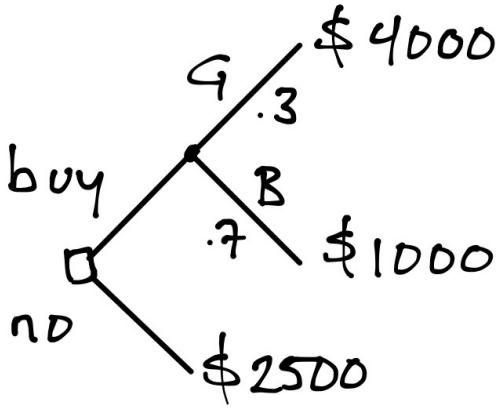
Price is $2500

Value depends on condition:

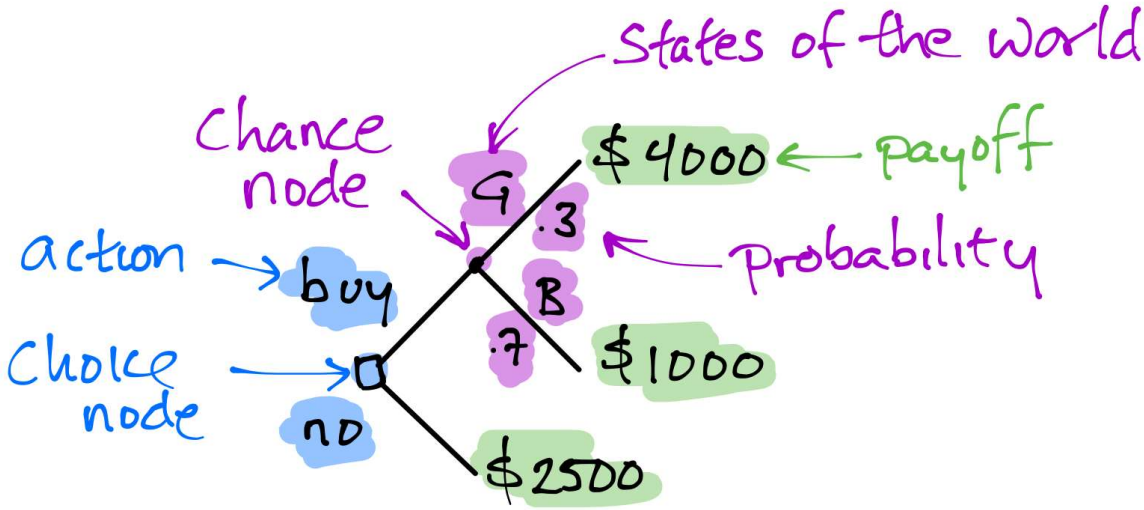
Condition	Value	Probability
Good (G):	$4000	30%
Bad (B):	$1000	70%

Decision: buy or not?

Step 1: draw decision tree from *left* to *right* in causality



Labeling the pieces:



Two kinds of nodes:

Choice => branches are **actions**

Chance => branches are **states** and have **probabilities**

Payoff types:

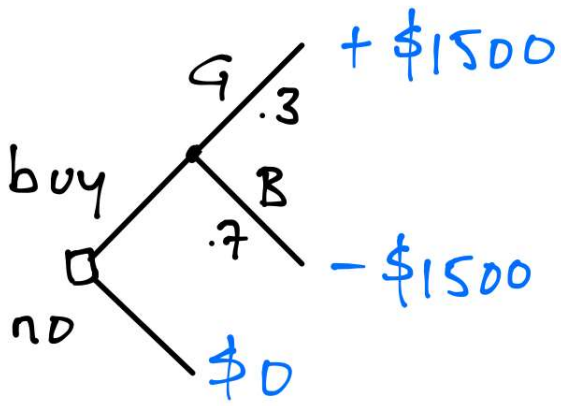
Gross: **actual value** under each condition (used above)

Net: **change** in value from BAU

Car tree with net payoffs:

Net if G: $\$4000 - \$2500 = +\$1500$

Net if B: $\$1000 - \$2500 = -\$1500$



Step 2: simplify the tree from *right* to *left*

Apply two rules repeatedly:

1. **Choice** node => take **action** with the **best payoff**
2. **Chance** node => replace with its **expected value**

Expected value is the probability-weighted average payoff

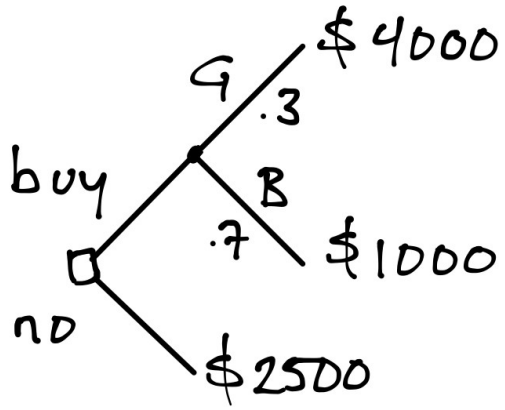
Defining variables:

N = number of states
ρ_i = probability of state i
X_i = payoff in state i

Expected value (EV) formula:

$$EV = \sum_{i=1}^{N} \rho_i X_i$$

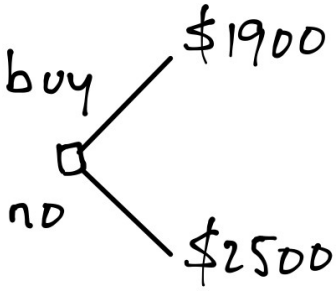
Applying to original tree:



EV of chance node:

$$EV = 0.3 * 4000 + 0.7 * 1000 = 1900$$

Replacing node with its EV simplifies the tree:



Take action with best payoff at choice node (2 pieces of information):

Action: No (don't buy)
Payoff: $2500

Example 2: Buying Information

Lemon Busters (LB) test

Cost: $400

Two possible reports:

"Car is good" => rG for short: "reports good"

"Car is bad" => rB for short: "reports bad"

Suppose LB never makes mistakes (infallible):

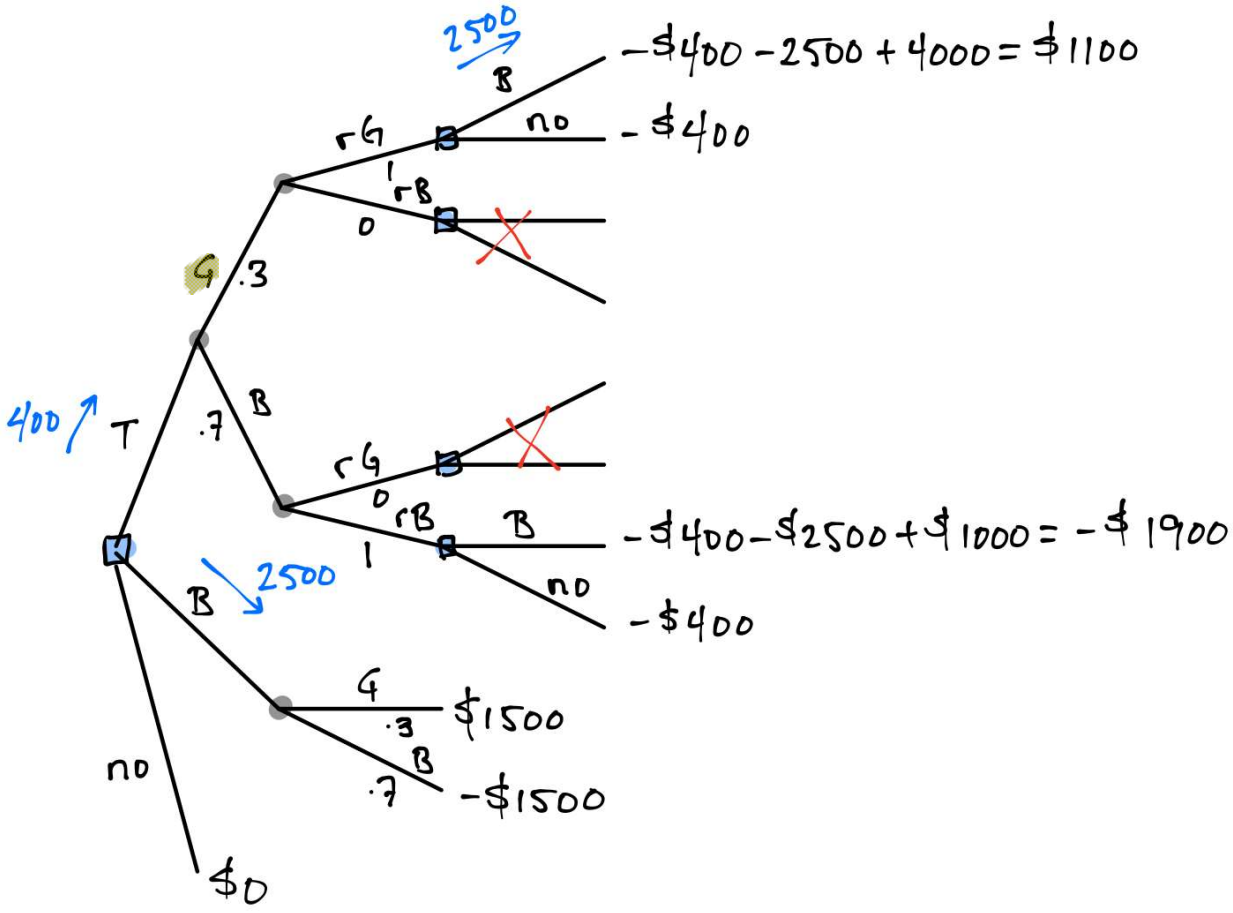
Car condition	Report rG	Report rB
G	100%	**0%**
B	**0%**	100%

Now two decisions:

Buy test? T

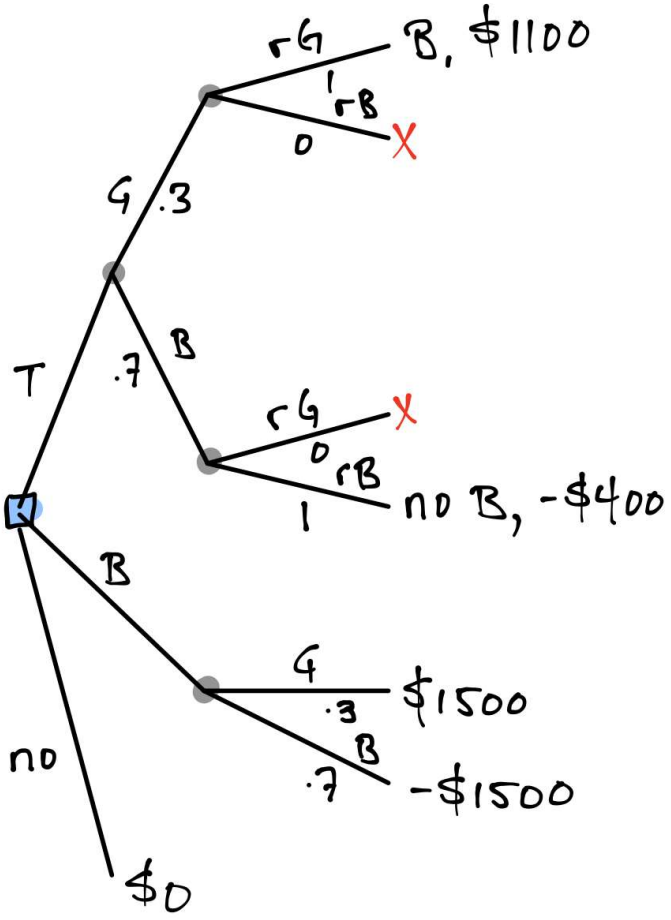
Buy car? B

New tree with net payoffs and action costs:



Evaluating:

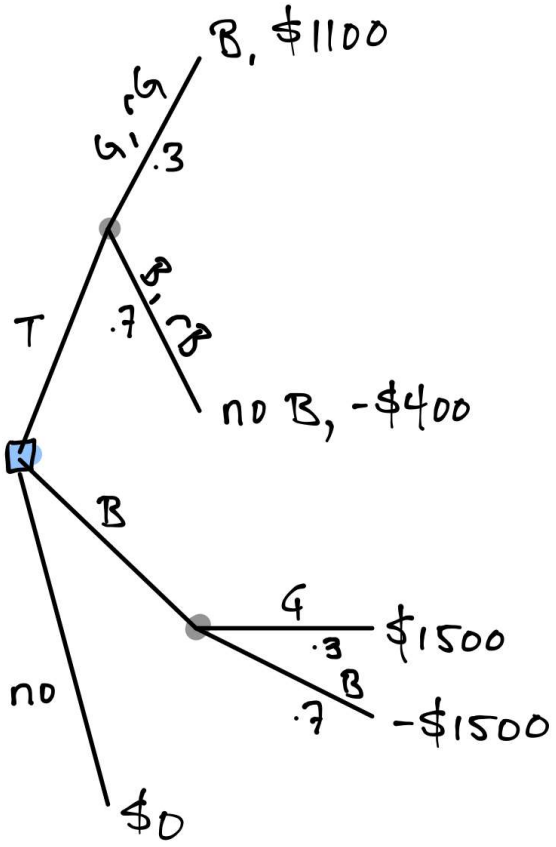
Right-most nodes are choices:



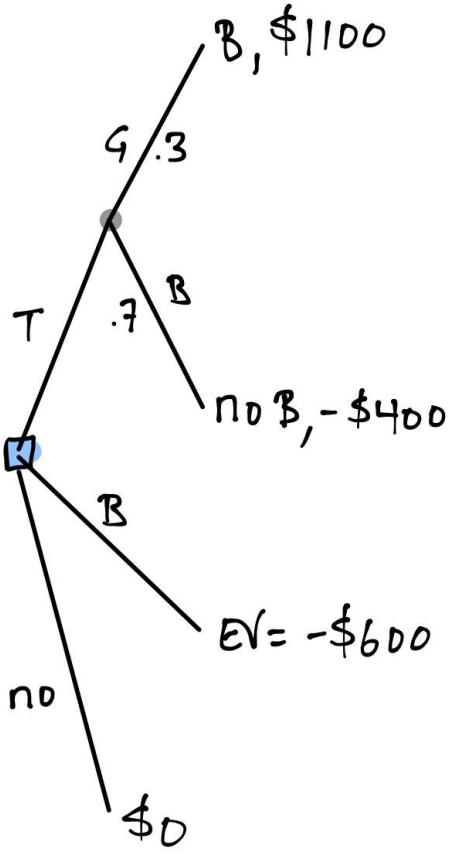
Choose B, $1100

Choose no B, $400

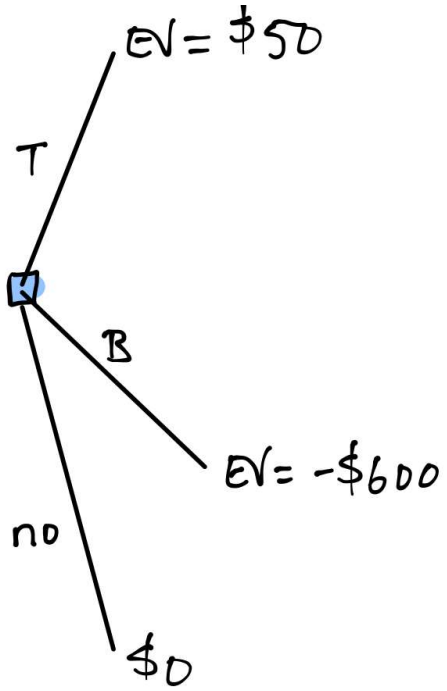
Simplify by removing the impossible branches:



Now right-most nodes are chance nodes:



EV = 0.3*(1500)+0.7*(-1500)
EV = -600



EV = 0.3*(1100)+0.7*(-400)
EV = 50

Finally, remaining node is a choice:

Best option: T, EV = $50

Information worth more than $400

Daily exercise on Google Classroom