Production

Two fundamental activities:

1. Cost minimization

    For each possible output Q find the cheapest way to produce it

    Deeply similar to household choice
        Choosing inputs subject to a constraint

    Produces a menu of options:

Q	Total cost
1	$100
2	$190
3	$270

2. Output decision

    Choose output given costs, market conditions, and producer's goals
        Which option from the menu of possible Qs?

Cost Minimization

Mirror image of the individual choice model

Deeply analogous, but with slight changes

Same four conceptual components:

1. Set of options available
   What is the choice over?
   Now: bundles of **inputs** or **factors of production**

2. Feasibility
   What can the decision maker actually do?
   Now: must choose bundle **sufficient to make target Q**

1. Ranking
   How does the decision maker feel about the options?
   Now: wants **minimize cost**

2. Choice
   What does the decision maker choose?
   Now: **cost-minimizing input bundle**

Example: custom sailboat hulls

1. Options available

   Inputs:

   Labor time, building space, tools, molds, resin, fiberglass, …

Simplify to two:
Labor (L) and capital (K)

Input bundles are points in the L, K plane:

2. Feasibility

Which bundles are enough to produce a target Q?

Depends on process used to make the product
Summarize using a *production function*:

$Q(L, K)$

Suppose for sailboats the function is:

$$Q = \frac{1}{18} K^{0.5} L^{0.5}$$

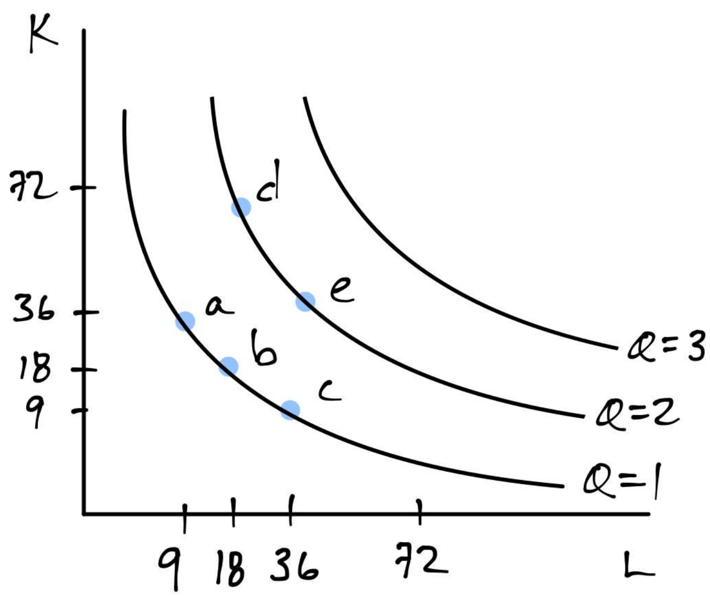
Example of a Cobb-Douglas production function

Translating example bundles of inputs to output:

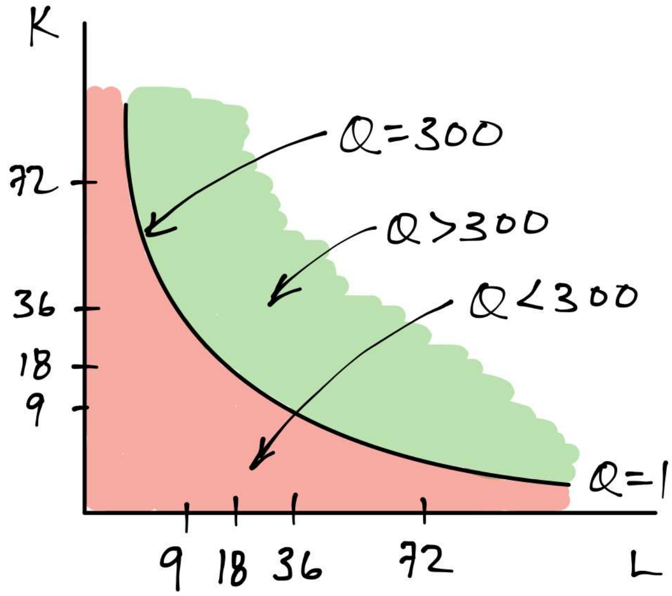
Bundle	L	K	Q

a	9	36	$\frac{9^{0.5} * 36^{0.5}}{18} = 1$
b	18	18	$\frac{18^{0.5} * 18^{0.5}}{18} = 1$
c	36	9	$\frac{36^{0.5} * 9^{0.5}}{18} = 1$
d	18	72	$\frac{18^{0.5} * 72^{0.5}}{18} = 2$
e	36	36	$\frac{36^{0.5} * 36^{0.5}}{18} = 2$

Bundles that produce the same output are *isoquants*:



Look like ICs but actually define feasible sets:



Must be on or above isoquant to produce Q=1

3. Ranking

What do the bundles cost?

Accounting equation for total cost (TC) is sum of spending on each input:

$$TC = P_K K + P_L L$$

Can graph bundles adding up to any given total cost

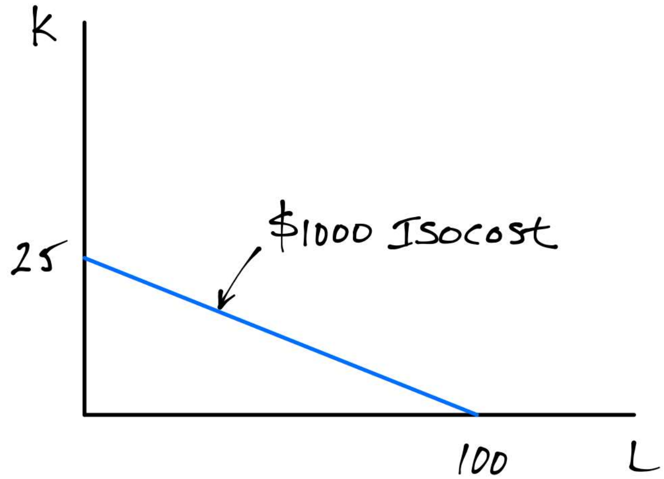
Sailboat example:

Suppose prices are $P_L = 10$ and $P_K = 40$

Which bundles have $TC = \$1000$?

$$TC = P_K K + P_L L$$

$$\$1000 = \$40 * K + \$10 * L$$



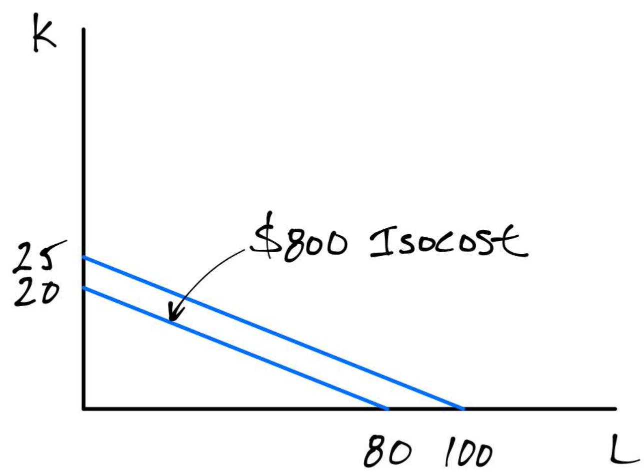
K intercept:
1000/40 = 25

L intercept:
1000/10 = 100

Adding isocost for bundles where $TC = \$800$

$$TC = P_K K + P_L L$$

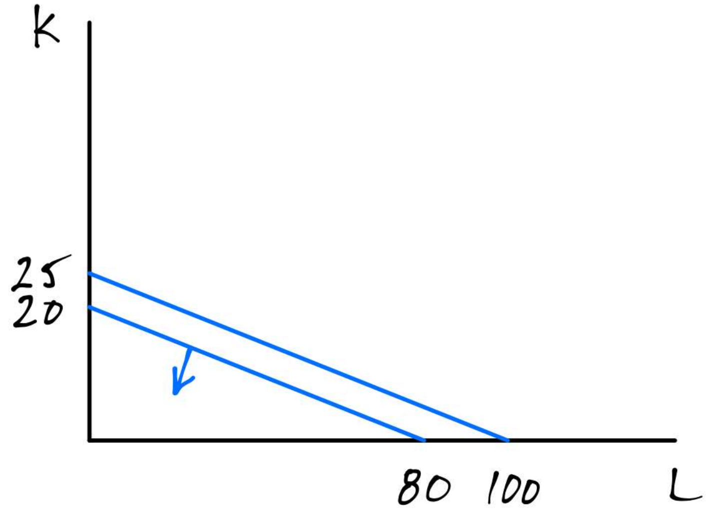
$$\$800 = \$40 * K + \$10 * L$$



K intercept:
800/40=20

L intercept:
800/10=80

Isocosts look like BCs but actually show preferences:

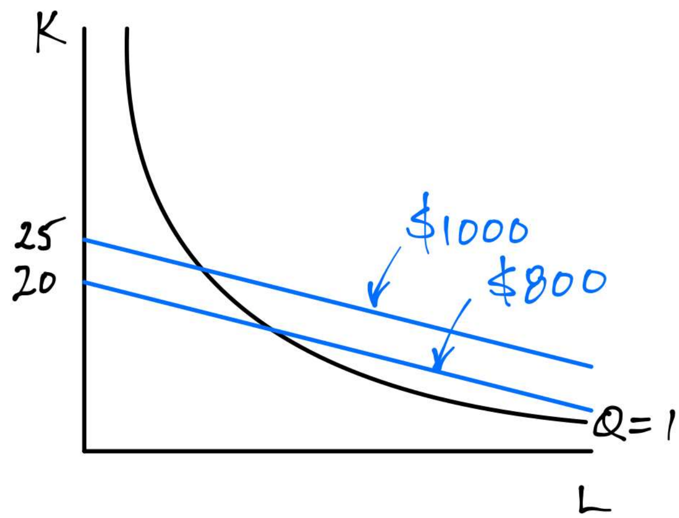


Preferred direction: toward origin

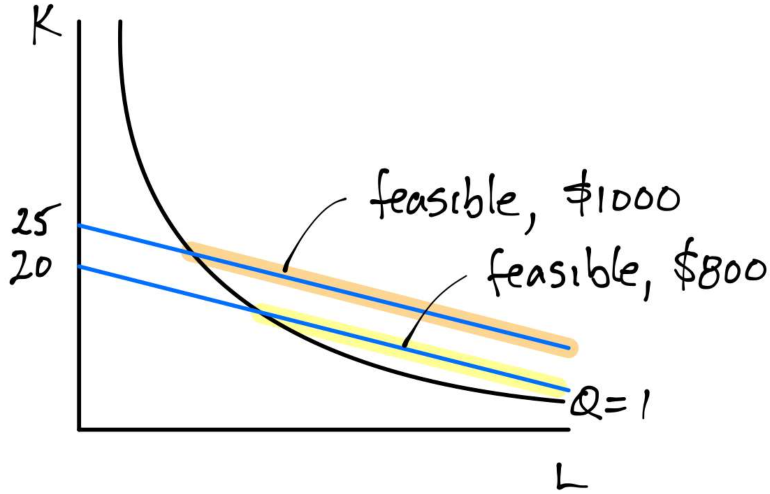
Best bundle will be on *innermost feasible isocost*

4. Choice

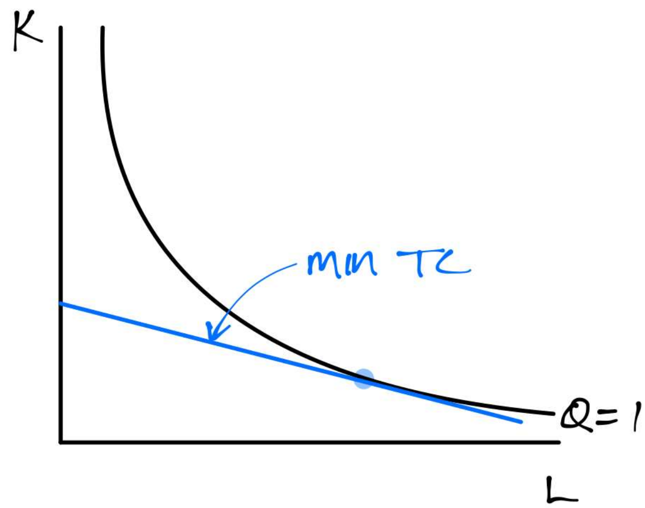
Overlay isocost (preferences) on isoquant (constraint):



Comparing costs of feasible bundles:

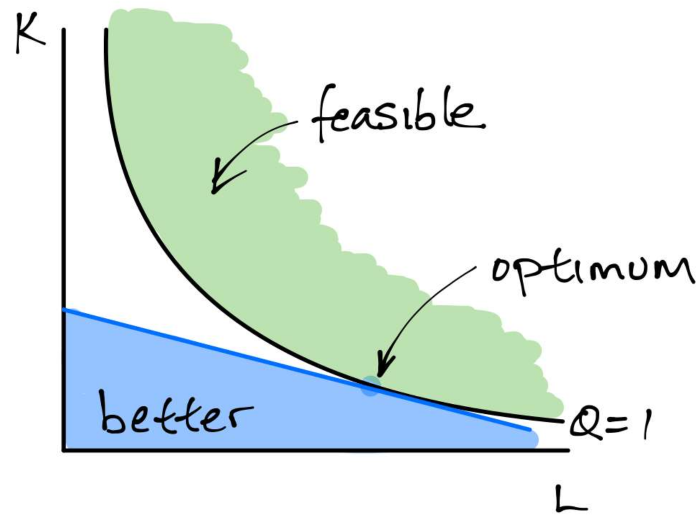


Minimum cost bundle is on innermost TC curve:



Like individual choice:
curves just touch

Optimum input bundle is **exactly analogous** to individual choice:



Other feasible bundles are **worse** (expensive)

Better bundles (cheaper) are **not feasible**

Like individual choice, can derive **demand equations** for inputs:

Household		Production
$Q_X(M, P_X, P_Y)$		$L(Q, P_L, P_K)$
$Q_Y(M, P_X, P_Y)$		$K(Q, P_K, P_L)$

Here: called input or factor demands

For this production function, can show factor demands are:

$$K = 18Q\left(\frac{P_L}{P_K}\right)^{0.5}$$

$$L = 18Q\left(\frac{P_K}{P_L}\right)^{0.5}$$

Factor demands are the solution to the cost minimization problem:
Show least cost mix of inputs for any Q

Example applications:

Case 1

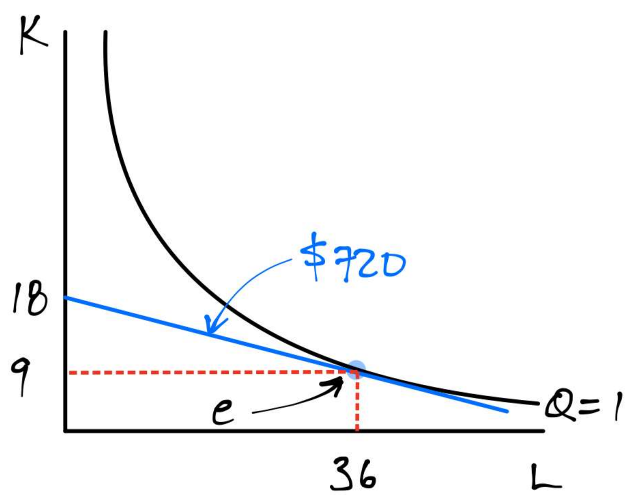
Target: Q=1

Prices: $P_L = 10, P_K = 40$

$$K = 18 * 1 * \left(\frac{10}{40}\right)^{0.5} = 9$$

$$L = 18 * 1 * \left(\frac{40}{10}\right)^{0.5} = 36$$

$$TC = 40 * 9 + 10 * 36 = 720$$



Case 2

Target: Q=2

Prices: Same as above

$$K = 18 * 2 * \left(\frac{10}{40}\right)^{0.5} = 18$$

$$L = 18 * 2 * \left(\frac{40}{10}\right)^{0.5} = 72$$

$$TC = 1440$$

Case 3

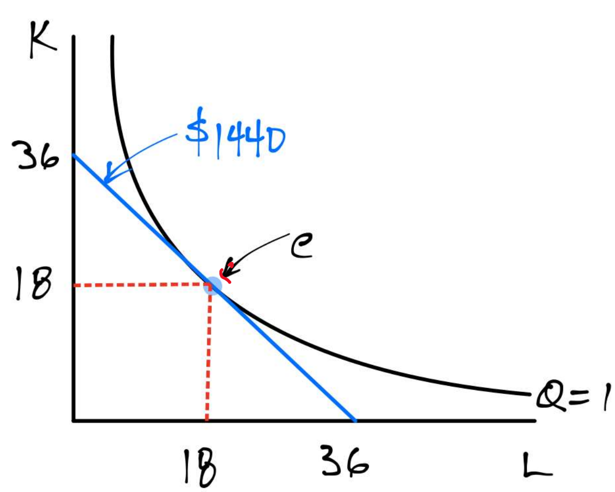
Target: Q=1

Prices: Higher P_L = 40, same $P_K = 40$

$$K = 18 * 1 * \left(\frac{40}{40}\right)^{0.5} = 18$$

$$L = 18 * 1 * \left(\frac{40}{40}\right)^{0.5} = 18$$

$$TC = 40 * 18 + 40 * 18 = 1440$$



For other production functions:

Similar procedure but different factor demands