Cost Functions

Can summarize producer's options by deriving several cost functions

Total cost	$TC(Q)$	Total cost of producing Q units
Average cost	$AC(Q)$	Average cost per unit
Marginal cost	$MC(Q)$	Cost of producing one additional unit

Total cost (TC) function:

Total cost of producing Q units in the minimum cost way

Analogous to the expenditure function:
- Substitute factor demands into the accounting equation for cost
- Result gives the minimum cost of reaching any given Q

Using the sailboat example:

Accounting equation for costs:

$$TC = P_K K + P_L L$$

Factor demands:

$$K = 18Q\left(\frac{P_L}{P_K}\right)^{0.5}$$

$$L = 18Q\left(\frac{P_K}{P_L}\right)^{0.5}$$

Substituting:

$$TC = P_K * 18Q\left(\frac{P_L}{P_K}\right)^{0.5} + P_L * 18Q\left(\frac{P_K}{P_L}\right)^{0.5}$$

Simplifying gives the *total cost function* for this producer:

$$TC = 36Q(P_K)^{0.5}(P_L)^{0.5}$$

$$TC(Q, P_K, P_L)$$

Simplify further by inserting prices; suppose $P_K = 40$, $P_L = 10$

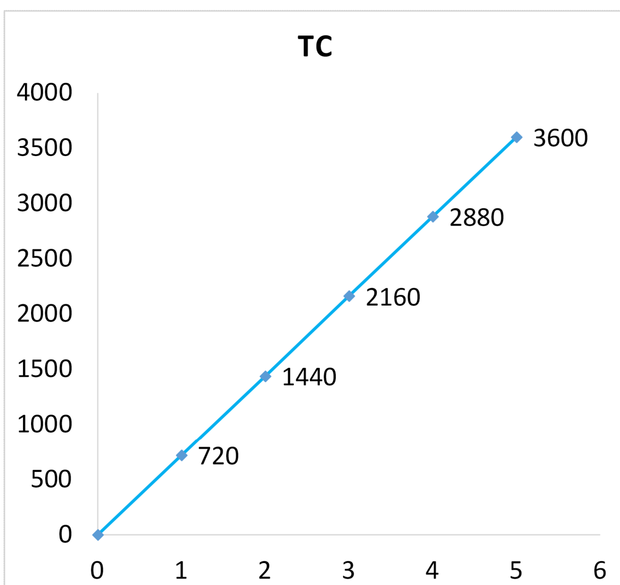
$$TC = 36Q(40)^{0.5}(10\,)^{0.5}$$

$$TC = 720Q$$

Check against previous results:

$$Q = 1, TC = 720$$
$$Q = 2, TC = 1440$$

Plotting:



Average cost (AC) function:

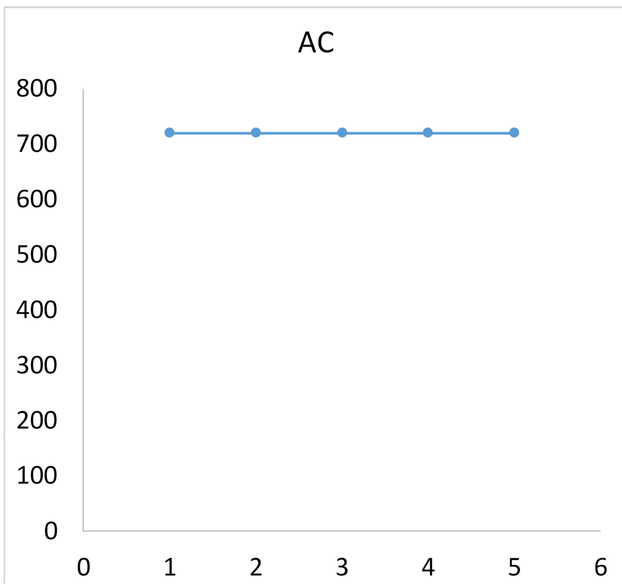
Average cost per unit

Definition:

$$AC(Q) = \frac{TC(Q)}{Q}$$

Sailboat example:

$$AC(Q) = \frac{720Q}{Q} = 720$$



Flat AC curve indicates production has *constant returns to scale (CRTS)*

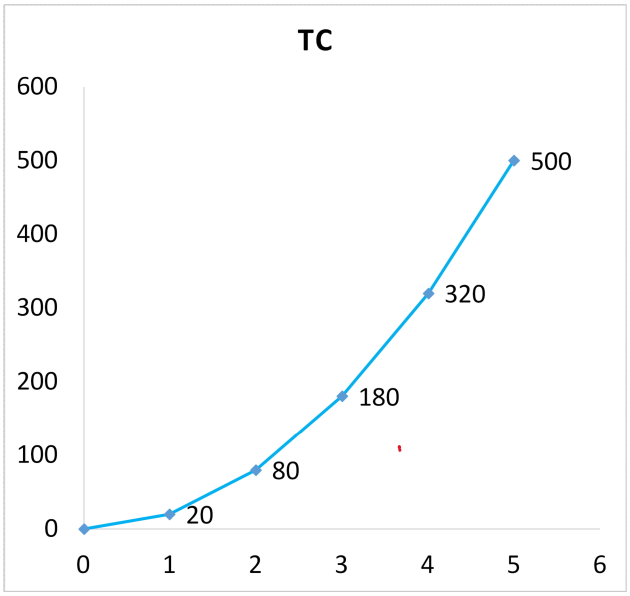
Can increase or decrease Q without changing average cost

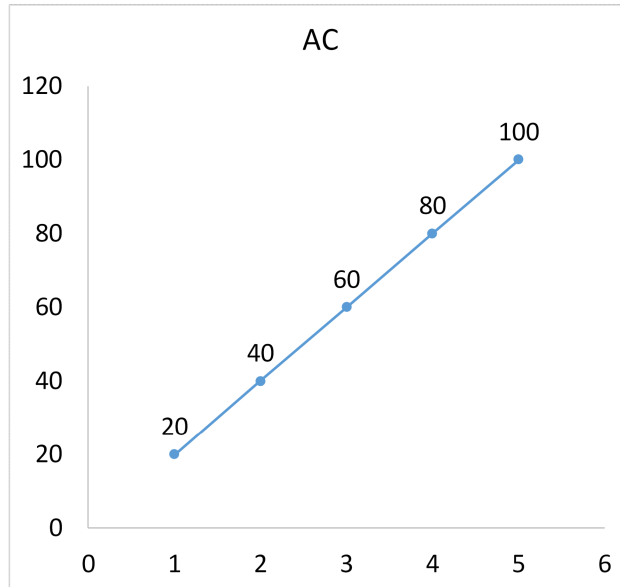
If AC rises with Q, production has *decreasing returns to scale*

Example:

$$TC = 20Q^2$$

$$AC = \frac{20Q^2}{Q} = 20Q$$





If *total costs* rise *rapidly* as Q increases:

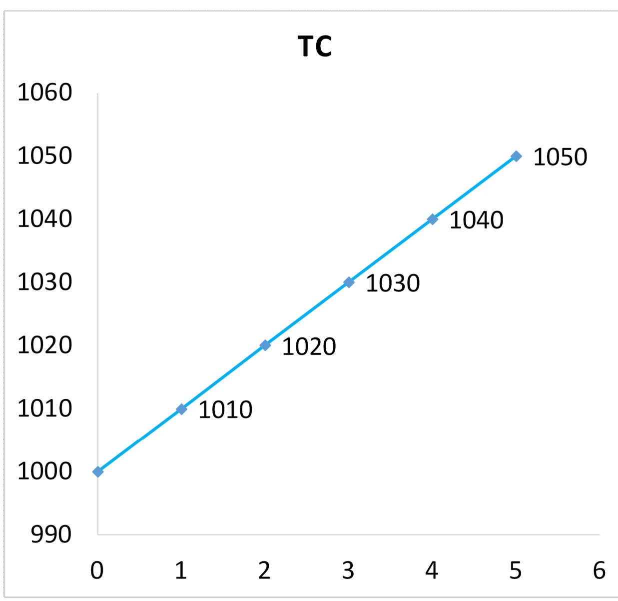
Cost per unit (AC) rises

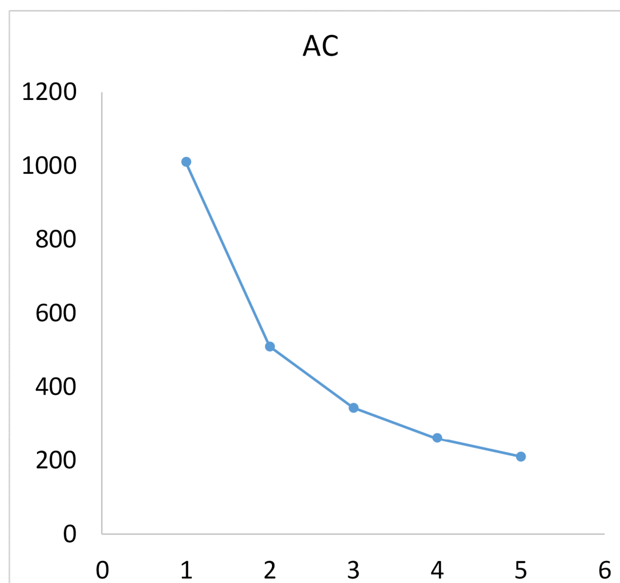
If AC falls with Q, production has *increasing returns to scale*

Example:

$$TC = 1000 + 10Q$$

$$AC = \frac{1000}{Q} + 10$$





If *total costs* rise *slowly* as Q increases:

Cost per unit (AC) falls

Typical of software or pharmaceuticals:
Products involving a lot of intellectual property (IP)

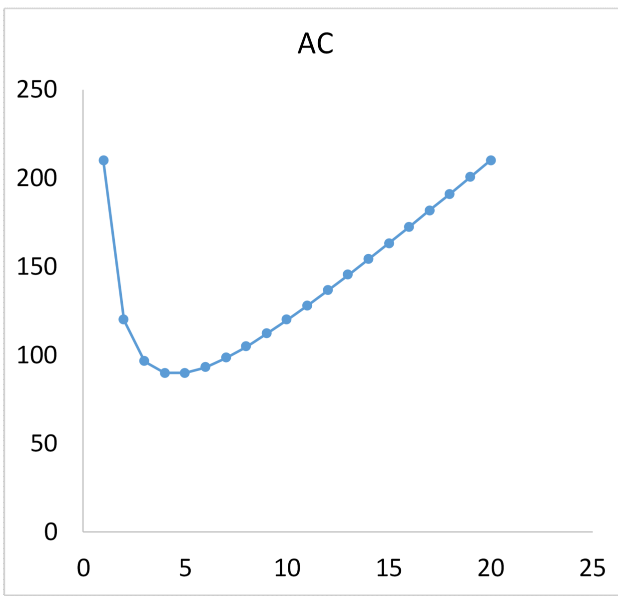
Note TC > 0 even when Q = 0
$1000 known as a *fixed cost*

Many producers have all three kinds of return at different Qs:

Example:

$$TC = 200 + 10Q^2$$

$$AC = \frac{200}{Q} + 10Q$$



Low Q:
Increasing returns

Middle Q:
Roughly CRTS

Large Q:
Decreasing returns

- Fixed cost of $200 creates initial increasing returns
- Eventual decreasing returns due to Q^2 in TC

Marginal cost (MC) function:

Cost of producing one additional unit beyond current Q

Definition:

$$MC(Q) = \frac{\Delta TC}{\Delta Q}$$

Sailboat example:

Start at Q and increase output to $Q + 1$:

$$MC(Q) = \frac{TC(Q+1) - TC(Q)}{1}$$

Calculating TCs:

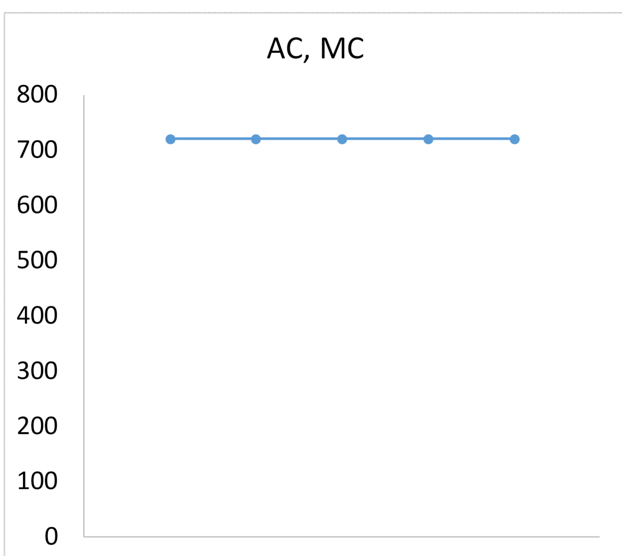
$$TC(Q) = 720Q$$

$$TC(Q+1) = 720(Q+1) = 720Q + 720$$

Solving for MC:

$$MC(Q) = \frac{720Q + 720 - 720Q}{1} = 720$$

Identical to AC because of CRTS

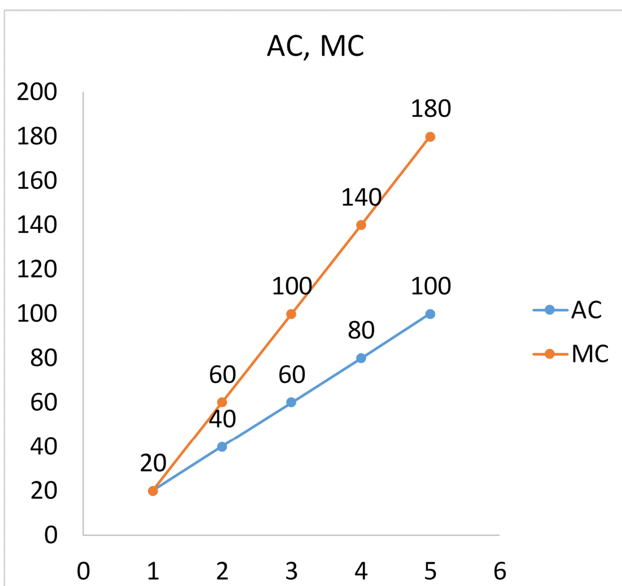


Decreasing returns example:

$$TC = 20Q^2$$

Q	TC	AC	$\Delta TC/\Delta Q$	MC
0	0			
1	20	20	(20-0)/1	20
2	80	40	(80-20)/1	60
3	180	60	(180-80)/1	100
4	320	80	(320-180)/1	140

Graphing:



High cost of added units draws AC up

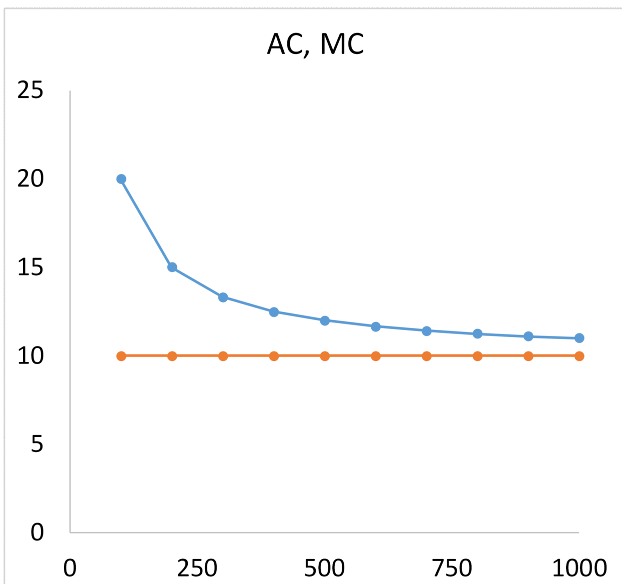
Increasing returns:

$$TC = 1000 + 10Q$$

$$AC = \frac{1000}{Q} + 10$$

$$MC = TC(Q+1) - TC(Q) = 10$$

Q	TC	AC	$\Delta TC/\Delta Q$	MC
0	1000			
100	2000	20	(2000-1000)/100	10
200	3000	15	(3000-2000)/100	10
300	4000	13.3	(4000-3000)/100	10



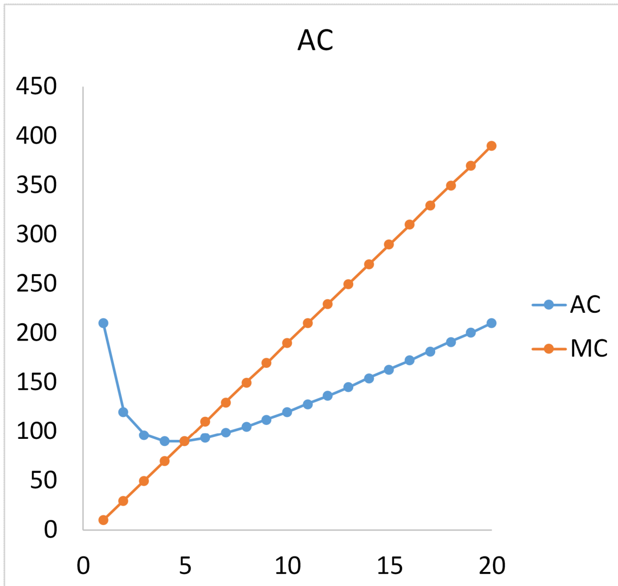
Unit cost gradually falls toward MC

Typical case for many producers:

$$TC = 200 + 10Q^2$$

Q	TC	AC	$\Delta TC/\Delta Q$	MC
0	200			
1	210	210	(210-200)/1	10
2	240	120	(240-210)/1	30
3	290	96.7	(290-240)/1	50
4	360	90	…	70

5	450	90	…	90
6	560	93.3	…	110



MC below AC when returns are increasing (low Q)

MC equals AC in CRTS region (middle Q)

MC above AC when returns are decreasing (high Q)

General result:

MC crosses AC at minimum AC