

# Output Decision

Optimal Q depends on producer's goals

Two main options:

1. Maximize *output* (without running a deficit)
  2. Maximize *profits*
- Can do one or the other but not both simultaneously
  - To implement, need to consider *revenue* and *profit*

Revenue equations:

Total revenue, TR:

Definition:

$$TR = P * Q$$

Average revenue, AR:

Definition:

$$AR = \frac{TR}{Q}$$

Simplifying:

$$AR = \frac{P * Q}{Q}$$

$$AR = P \quad \text{Average revenue} = P$$

## Marginal revenue, MR:

Definition:

$$MR = \frac{\Delta TR}{\Delta Q}$$

# Profits and Output Goals

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## Definition of profit:

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Will use  $\pi$  to represent profit in equations

### (1) Total profit, $T\pi$ :

Definition:

$$T\pi = TR - TC$$

Total profit = total revenue - total cost

### (2) Average profit, $A\pi$ :

Definition:

$$A\pi = \frac{T\pi}{Q}$$

Simplifying:

$$A\pi = \frac{TR - TC}{Q}$$

$$A\pi = \frac{TR}{Q} - \frac{TC}{Q}$$

$$A\pi = AR - AC$$

$$A\pi = P - AC$$

Average profit = price - AC

### (3) Marginal profit, $M\pi$ :

### Definition:

$$M_{\pi} = \frac{\Delta T_{\pi}}{\Delta Q}$$

Simplifying:

$$M_{\pi} = \frac{\Delta TR - \Delta TC}{\Delta Q}$$

$$M_{\pi} = \frac{\Delta TR}{\Delta Q} - \frac{\Delta TC}{\Delta Q}$$

$$M_{\pi} = MR - MC$$

Marginal profit = MR - MC

Can use profit equations to find **optimal Q**

Goal 1: maximizing output:

Raise Q as long as profit isn't negative:

**1. Increase Q** when:    Profit > 0     or     $A\pi(Q) > 0$   
                                          $P - AC > 0$   
                                          $P > AC$

2. Stop at Q where:  $\text{Profit} = 0$  or  $P = AC$

Goal 2: maximize profit:

Raise Q as long as doing so raises profit:

1. *Increase Q* when: Profit rising      or       $M\pi(Q) > 0$   
 $MR - MC > 0$   
 $MR \geq MC$

2. *Stop* at Q where: Profit maximum or  $MR = MC$

# Demand and Producer's Choice of Price

What should the producer charge for the good?

Depends on the *degree of competition* in the market

Often called the "market environment"

Four broad cases:

1. Monopoly  
No competitors
2. Perfect competition  
Many competitors  
Identical products
3. Monopolistic competition  
Many competitors  
Differentiated products
4. Oligopoly  
Small number of competitors  
Strategic behavior arises

This class focuses on 1 and 2

Key difference between cases:

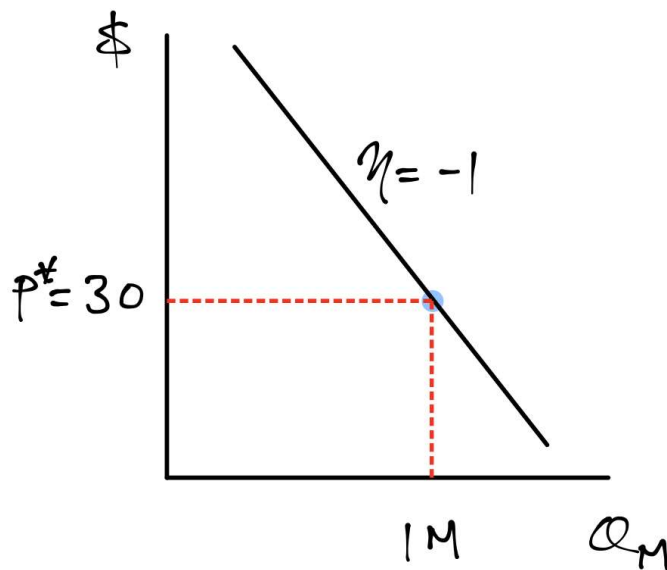
Elasticity of demand faced by the producer

## Example: alpaca wool

Market P: \$30

Market Q: 1M pounds

Elasticity: -1



### Case 1: Monopoly

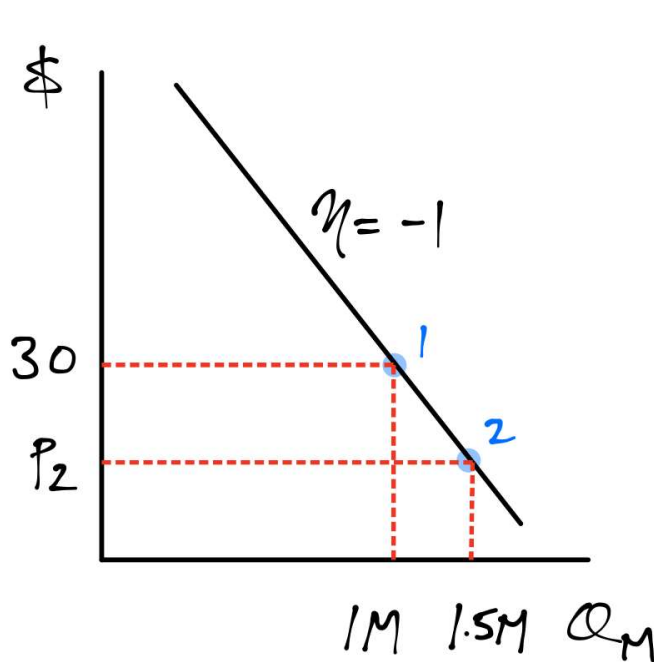
Producer sees the market demand:

$$WTP_M(Q) = P$$

Must adjust P when changing Q:

- Raising Q requires reducing P
- Raising P requires reducing Q

Example: suppose want to raise Q by 50% to 1.5M:



$$\eta = \frac{\% \Delta Q_M}{\% \Delta P_M}$$

$$\% \Delta P_M = \frac{50\%}{-1}$$

$$\% \Delta P_M = -50\%$$

$$P_2 = \$15$$

## Case 2: Perfect competition

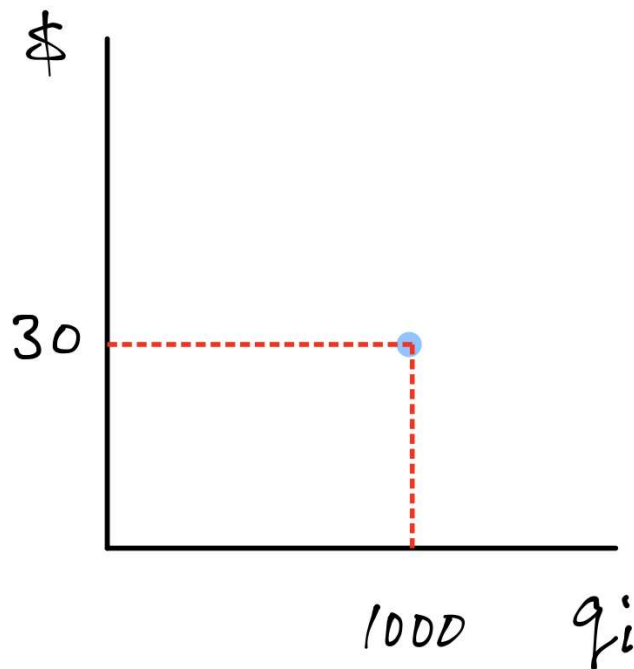
**Individual** producer sees **much** more elastic demand

Suppose:

- 1000 identical small producers
- Each produces  $q_i = 1000$

Start with one point on each producer's apparent demand:





Producer considers raising  $q_i$  by 50%: 500 lbs

Impact on market price?

Small percent change for the market:

$$\% \Delta Q_M = \frac{500}{1M} = 0.05\%$$

Finding the price change:

$$\eta = \frac{\% \Delta Q_M}{\% \Delta P_M}$$

$$\% \Delta P_M = \frac{\% \Delta Q_M}{\eta} = \frac{0.05\%}{-1} = -0.05\%$$

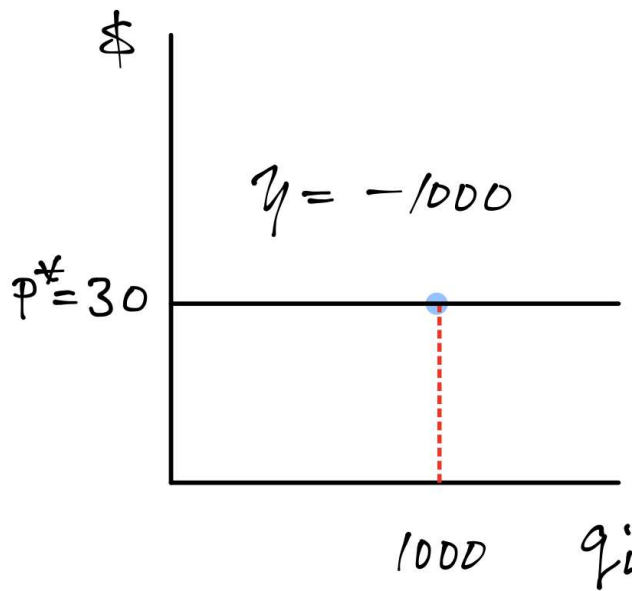
$$\Delta P_M = (-0.0005)(\$30) = -\$0.015$$

$$\Delta P_M = -1.5 \text{ ¢}$$

Apparent elasticity to the producer:

$$\eta_i = \frac{\% \Delta q_i}{\% \Delta P_M} = \frac{+50\%}{-0.05\%} = -1000$$

Graphing:



Flat for all practical purposes:

Producers can act as though the price is fixed at  $P^*$   
Known as *price taking* behavior

## Output Decision Example

Can now find optimal output for four cases:

|            |                   | Market type |             |
|------------|-------------------|-------------|-------------|
|            |                   | Competitive | Monopoly    |
| Goal       |                   | $P = P^*$   | $P = WTP_M$ |
| Output max | Q where $P = AC$  | Case 1      | Case 3      |
| Profit max | Q where $MR = MC$ | Case 2      | Case 4      |

Example: sailboat hulls

$$TC = 1800 + 72 * Q^2$$

Monopoly demand:  $WTP = 4000 - 300Q$

Competitor's demand:  $P^* = 1200$

Can find optimal Q with a spreadsheet with 5 columns:

1.  $Q$
2.  $P(Q)$
3.  $TR = P * Q$
4.  $TC = 1800 + 72 * Q^2$
5.  $\Pi = TR - TC$

Example: competitive producer with  $P=1200$

| Q | P    | TR | TC   | Profit |
|---|------|----|------|--------|
| 0 | 1200 | 0  | 1800 | -1800  |

|     |      |      |      |      |
|-----|------|------|------|------|
| 1   | 1200 | 1200 | 1872 | -672 |
| 2   | 1200 | 2400 | 2088 | 312  |
| ... |      |      |      | ...  |

Spreadsheet on web site

# Snapshots of Spreadsheets

e075

## Competitive case:

### Output Decision

Short Run with Capital Stock Fixed

Linear Demand Curve:  $P = A - B \cdot Q$   
Total Cost Equation:  $TC = 1800 + 72 \cdot Q^2$

|   |       |
|---|-------|
| A | 1,200 |
| B | 0     |

|            | Q  | Price | TR     | TC     | Profit  | AC    | AR    | AR-AC | MC    | MR    | MR-MC  |
|------------|----|-------|--------|--------|---------|-------|-------|-------|-------|-------|--------|
|            | 0  | 1,200 | 0      | 1,800  | -1,800  | --    | --    | --    | --    | --    | --     |
|            | 1  | 1,200 | 1,200  | 1,872  | -672    | 1,872 | 1,200 | -672  | 72    | 1,200 | 1,128  |
|            | 2  | 1,200 | 2,400  | 2,088  | 312     | 1,044 | 1,200 | 156   | 216   | 1,200 | 984    |
|            | 3  | 1,200 | 3,600  | 2,448  | 1,152   | 816   | 1,200 | 384   | 360   | 1,200 | 840    |
|            | 4  | 1,200 | 4,800  | 2,952  | 1,848   | 738   | 1,200 | 462   | 504   | 1,200 | 696    |
|            | 5  | 1,200 | 6,000  | 3,600  | 2,400   | 720   | 1,200 | 480   | 648   | 1,200 | 552    |
|            | 6  | 1,200 | 7,200  | 4,392  | 2,808   | 732   | 1,200 | 468   | 792   | 1,200 | 408    |
|            | 7  | 1,200 | 8,400  | 5,328  | 3,072   | 761   | 1,200 | 439   | 936   | 1,200 | 264    |
| Profit max | 8  | 1,200 | 9,600  | 6,408  | 3,192   | 801   | 1,200 | 399   | 1,080 | 1,200 | 120    |
|            | 9  | 1,200 | 10,800 | 7,632  | 3,168   | 848   | 1,200 | 352   | 1,224 | 1,200 | -24    |
|            | 10 | 1,200 | 12,000 | 9,000  | 3,000   | 900   | 1,200 | 300   | 1,368 | 1,200 | -168   |
|            | 11 | 1,200 | 13,200 | 10,512 | 2,688   | 956   | 1,200 | 244   | 1,512 | 1,200 | -312   |
|            | 12 | 1,200 | 14,400 | 12,168 | 2,232   | 1,014 | 1,200 | 186   | 1,656 | 1,200 | -456   |
|            | 13 | 1,200 | 15,600 | 13,968 | 1,632   | 1,074 | 1,200 | 126   | 1,800 | 1,200 | -600   |
|            | 14 | 1,200 | 16,800 | 15,912 | 888     | 1,137 | 1,200 | 63    | 1,944 | 1,200 | -744   |
| Output max | 15 | 1,200 | 18,000 | 18,000 | 0       | 1,200 | 1,200 | 0     | 2,088 | 1,200 | -888   |
|            | 16 | 1,200 | 19,200 | 20,232 | -1,032  | 1,265 | 1,200 | -65   | 2,232 | 1,200 | -1,032 |
|            | 17 | 1,200 | 20,400 | 22,608 | -2,208  | 1,330 | 1,200 | -130  | 2,376 | 1,200 | -1,176 |
|            | 18 | 1,200 | 21,600 | 25,128 | -3,528  | 1,396 | 1,200 | -196  | 2,520 | 1,200 | -1,320 |
|            | 19 | 1,200 | 22,800 | 27,792 | -4,992  | 1,463 | 1,200 | -263  | 2,664 | 1,200 | -1,464 |
|            | 20 | 1,200 | 24,000 | 30,600 | -6,600  | 1,530 | 1,200 | -330  | 2,808 | 1,200 | -1,608 |
|            | 21 | 1,200 | 25,200 | 33,552 | -8,352  | 1,598 | 1,200 | -398  | 2,952 | 1,200 | -1,752 |
|            | 22 | 1,200 | 26,400 | 36,648 | -10,248 | 1,666 | 1,200 | -466  | 3,096 | 1,200 | -1,896 |
|            | 23 | 1,200 | 27,600 | 39,888 | -12,288 | 1,734 | 1,200 | -534  | 3,240 | 1,200 | -2,040 |

## Monopoly cases:

# Output Decision

Short Run with Capital Stock Fixed

Linear Demand Curve:  $P = A - B \cdot Q$

Total Cost Equation:  $TC = 1800 + 72 \cdot Q^2$

|   |       |
|---|-------|
| A | 4,000 |
| B | 300   |

|            | Q  | Price  | TR      | TC     | Profit   | AC    | AR     | AR-AC  | MC    | MR     | MR-MC   |
|------------|----|--------|---------|--------|----------|-------|--------|--------|-------|--------|---------|
|            | 0  | 4,000  | 0       | 1,800  | -1,800   | --    | --     |        | --    | --     |         |
|            | 1  | 3,700  | 3,700   | 1,872  | 1,828    | 1,872 | 3,700  | 1,828  | 72    | 3,700  | 3,628   |
|            | 2  | 3,400  | 6,800   | 2,088  | 4,712    | 1,044 | 3,400  | 2,356  | 216   | 3,100  | 2,884   |
|            | 3  | 3,100  | 9,300   | 2,448  | 6,852    | 816   | 3,100  | 2,284  | 360   | 2,500  | 2,140   |
|            | 4  | 2,800  | 11,200  | 2,952  | 8,248    | 738   | 2,800  | 2,062  | 504   | 1,900  | 1,396   |
| Profit max | 5  | 2,500  | 12,500  | 3,600  | 8,900    | 720   | 2,500  | 1,780  | 648   | 1,300  | 652     |
|            | 6  | 2,200  | 13,200  | 4,392  | 8,808    | 732   | 2,200  | 1,468  | 792   | 700    | -92     |
|            | 7  | 1,900  | 13,300  | 5,328  | 7,972    | 761   | 1,900  | 1,139  | 936   | 100    | -836    |
|            | 8  | 1,600  | 12,800  | 6,408  | 6,392    | 801   | 1,600  | 799    | 1,080 | -500   | -1,580  |
|            | 9  | 1,300  | 11,700  | 7,632  | 4,068    | 848   | 1,300  | 452    | 1,224 | -1,100 | -2,324  |
| Output max | 10 | 1,000  | 10,000  | 9,000  | 1,000    | 900   | 1,000  | 100    | 1,368 | -1,700 | -3,068  |
|            | 11 | 700    | 7,700   | 10,512 | -2,812   | 956   | 700    | -256   | 1,512 | -2,300 | -3,812  |
|            | 12 | 400    | 4,800   | 12,168 | -7,368   | 1,014 | 400    | -614   | 1,656 | -2,900 | -4,556  |
|            | 13 | 100    | 1,300   | 13,968 | -12,668  | 1,074 | 100    | -974   | 1,800 | -3,500 | -5,300  |
|            | 14 | -200   | -2,800  | 15,912 | -18,712  | 1,137 | -200   | -1,337 | 1,944 | -4,100 | -6,044  |
|            | 15 | -500   | -7,500  | 18,000 | -25,500  | 1,200 | -500   | -1,700 | 2,088 | -4,700 | -6,788  |
|            | 16 | -800   | -12,800 | 20,232 | -33,032  | 1,265 | -800   | -2,065 | 2,232 | -5,300 | -7,532  |
|            | 17 | -1,100 | -18,700 | 22,608 | -41,308  | 1,330 | -1,100 | -2,430 | 2,376 | -5,900 | -8,276  |
|            | 18 | -1,400 | -25,200 | 25,128 | -50,328  | 1,396 | -1,400 | -2,796 | 2,520 | -6,500 | -9,020  |
|            | 19 | -1,700 | -32,300 | 27,792 | -60,092  | 1,463 | -1,700 | -3,163 | 2,664 | -7,100 | -9,764  |
|            | 20 | -2,000 | -40,000 | 30,600 | -70,600  | 1,530 | -2,000 | -3,530 | 2,808 | -7,700 | -10,508 |
|            | 21 | -2,300 | -48,300 | 33,552 | -81,852  | 1,598 | -2,300 | -3,898 | 2,952 | -8,300 | -11,252 |
|            | 22 | -2,600 | -57,200 | 36,648 | -93,848  | 1,666 | -2,600 | -4,266 | 3,096 | -8,900 | -11,996 |
|            | 23 | -2,900 | -66,700 | 39,888 | -106,588 | 1,734 | -2,900 | -4,634 | 3,240 | -9,500 | -12,740 |