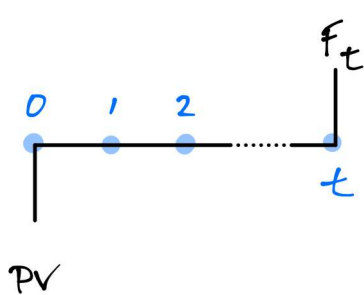


E: Present value (PV) refresher 1

Interpretation of PV:

- The amount of **money needed in a bank account today** (time 0) in order to **deliver a target sequence of payments** in the future.

Formula 1: Single payment:



$$PV = \frac{F_t}{(1 + r)^t}$$

Example 1: PV of a single payment:

F \$10k
t Year 5
r 10%

$$PV = \$10,000 / (1.1)^5 = \$6,209$$

Example 2: using PV as a benchmark for evaluating policies:

Proposed policy:

Cost \$3000 today
Delivers \$5000 in year 4

Cost of alternative using a bank account at $r=10\%$?

$$PV = \$5000 / (1.1)^4 = \$3415$$

Conclusion:

Project is \$415 cheaper than the bank.

Expressing via net present value (NPV):

$$NPV = PV \text{ of benefits} - PV \text{ of costs}$$

$$NPV = \$3415 - \$3000 = \$415$$

Project produces a net gain of \$415

Example 3: arbitrage trading:

Suppose know price of oil is rising:

$$P_0 = \$50$$

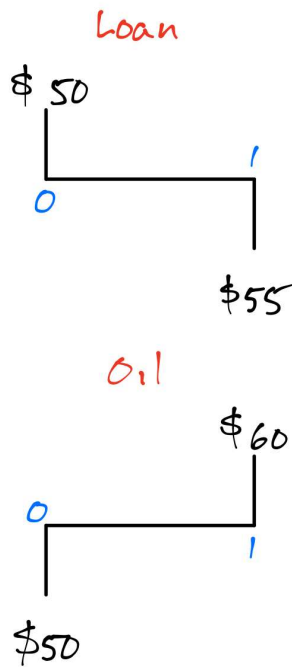
$$P_1 = \$60$$

$$r = 10\%$$

Arbitrage trade:

At t=0:

- Borrow \$50
- Buy oil



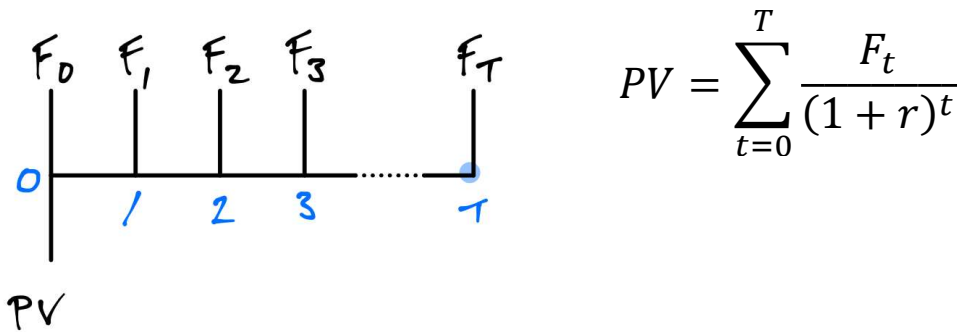
At t=1:

- Sell oil
- Repay loan

$$NPV = \$60/1.1 - \$55/1.1 = \$4.55$$

Profitable: returns exceed interest cost

Formula 2: extension to streams with multiple payments from 0 to T :

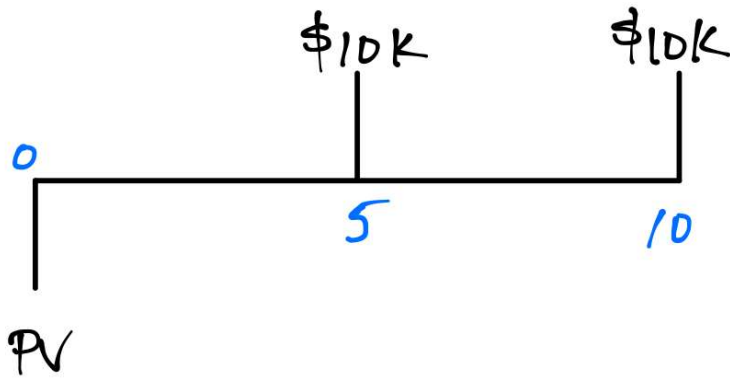


$$PV = \sum_{t=0}^T \frac{F_t}{(1+r)^t}$$

PV of the stream is the sum of the individual PVs

Example 4: two payments

Payments each \$10,000
One in year 5, one in year 10
 $r = 5\%$



$$PV = \$10,000/1.05^5 + \$10,000/1.05^{10} = \$13,974$$