E: Conditional probabilities

Application: the *Monty Hall* problem

From a long-running game show: *Let's Make a Deal*
Possibly best illustration *ever* of conditional probabilities

Situation:

- Contestant chooses a door
- Monty opens one of the others: no prize
- Asks the contestant if they'd like to switch

Intuitive reaction:

- No reason to change: the prize hasn't moved
- Must now have 50/50 chance

Flat wrong:

- Overlooks the fact that Monty's choice is **not random**
- **Never** opens the door with the prize
- As a result, choice reveals information

Analysis:

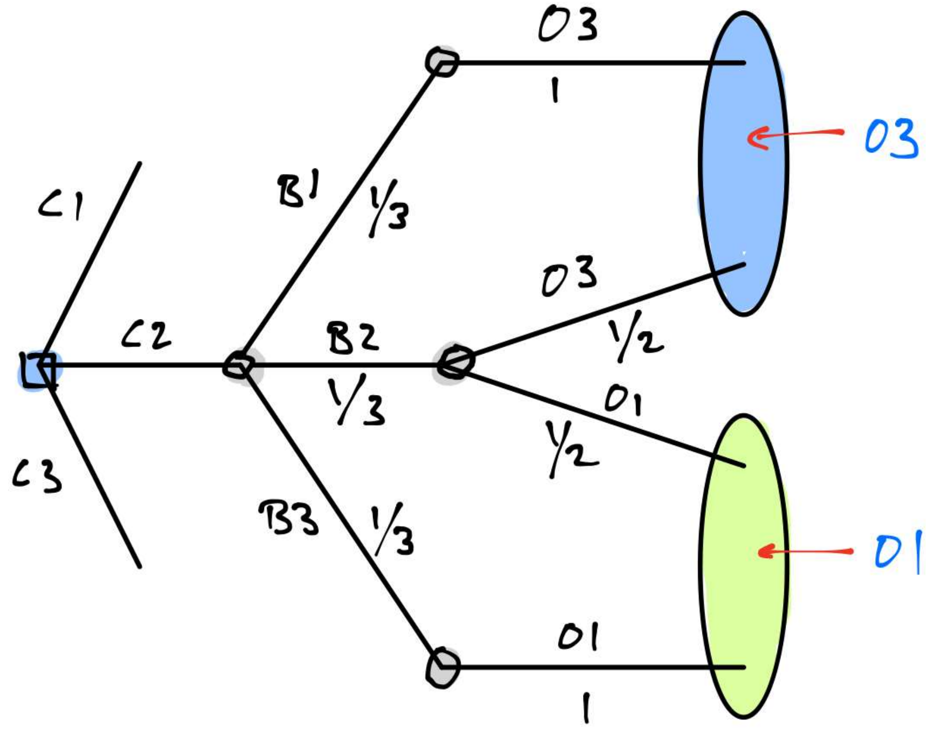
Notation:

Contestant's initial actions:
C1, C2, C3 = **choose** door 1, 2 or 3

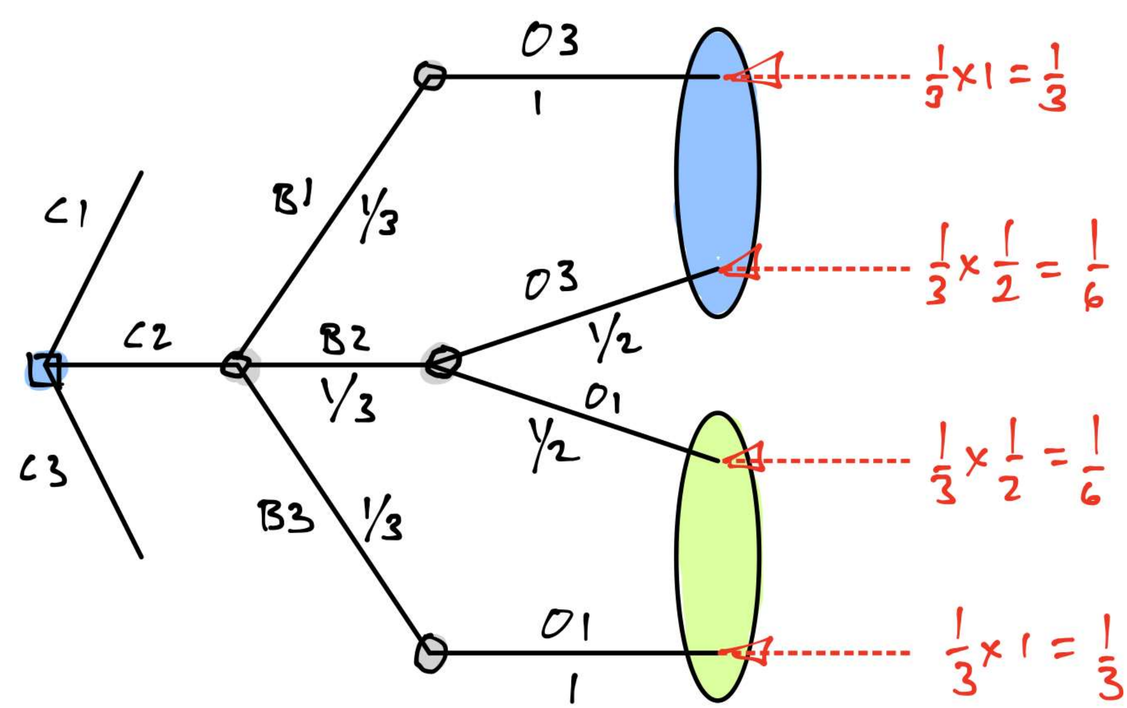
State: true location of the prize:
B1, B2, B3 = **behind** door 1, 2 or 3

Monty's actions:
O1, O2, O3 = **open** door 1, 2 or 3

Tree if contestant chooses 2:



Computing the **unconditional** probabilities:



Computing the probabilities of O3 and O1:

Prob of O3 = 1/3 + 1/6 = 0.5
Prob of O1 = 1/6 + 1/3 = 0.5

Computing the **conditional** probabilities of B2 and B1 for **O3**:

Probability behind 2 (B2) given O3: $\frac{1/6}{1/2} = \frac{1}{3}$
Sticking with current door: chance of car = 33%

Probability behind 1 (B1) given O3: $\frac{1/3}{1/2} = \frac{2}{3}$
Switching to door 1: chance of car = 67%

Conclusion:

Dramatically better to switch: probability is **double**
Monty's choice reveals very valuable information

Exercise on GC