E: Efficient incentive design, part 1

Principal-agent (PA) version of the biofuels startup

Participants:

Founder (F):	Has **idea** but no cash
Venture capitalist (VC):	Has **cash** but no idea

Payoffs:

Success (S):	$1M
Failure (F):	$10k
No startup (N):	$100k

PA changes:

1. Founder's effort (E) affects chance of success (S) but is costly to F

Level of effort	Cost to F	Prob of S
High (H):	$5k	20%
Low (L):	$2k	15%

1. VC only observes outcome (S or F), not E

Initially: F is risk neutral

What's the efficient level of effort?

Overall EV of startup, in thousands:

H: 0.2*(1000-5)+0.8*(10-5) = 203
L: 0.15*(1000-2) + 0.85*(10-2) = 156.5

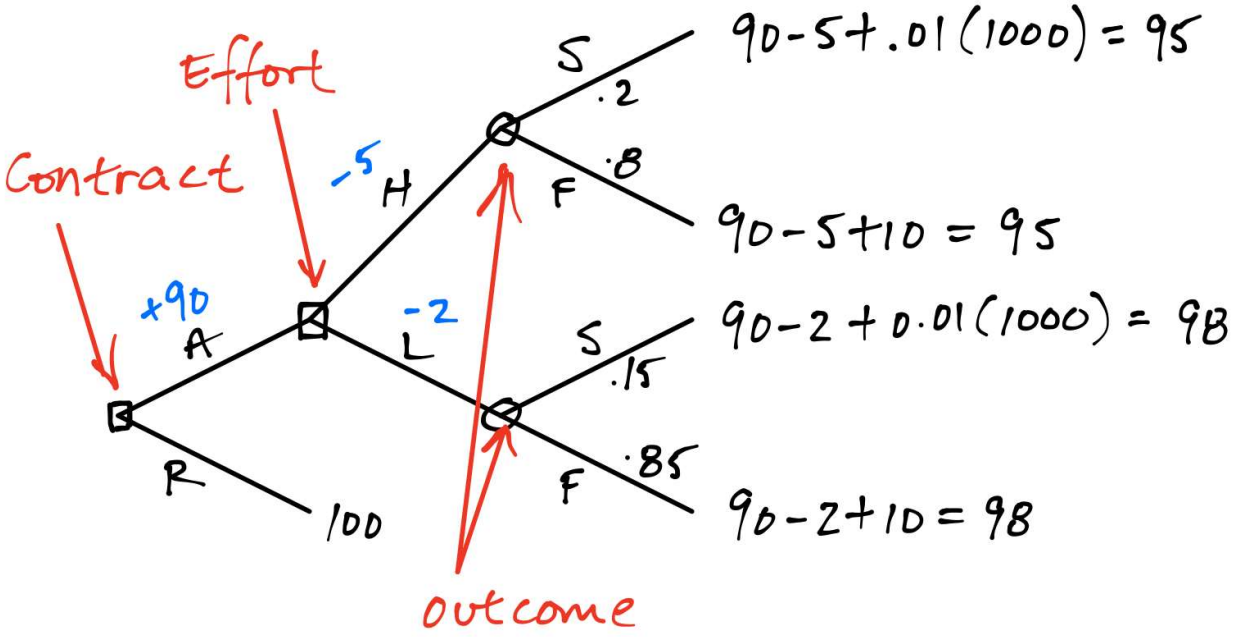
Conclusion: efficient effort is H

Case 1: VC offers previous $90k/99% contract

Two parameters:

Fixed payment (**Fx**):	$90k	VC pays to F
Share of ownership (**Sh**):	1%	Retained by F

Founder's tree with payoffs in thousands:



Contract decision: A = accept, R = reject
Effort decision: H = high, L = low

Outcome: S = success, F = failure

F's payoffs from **effort** choice:

$EV_H =$ 0.2*95 + 0.8*95 = 95
$EV_L =$ 0.15*98 + 0.85*98 = **98** Inefficient

F's payoffs from **contract** choice:

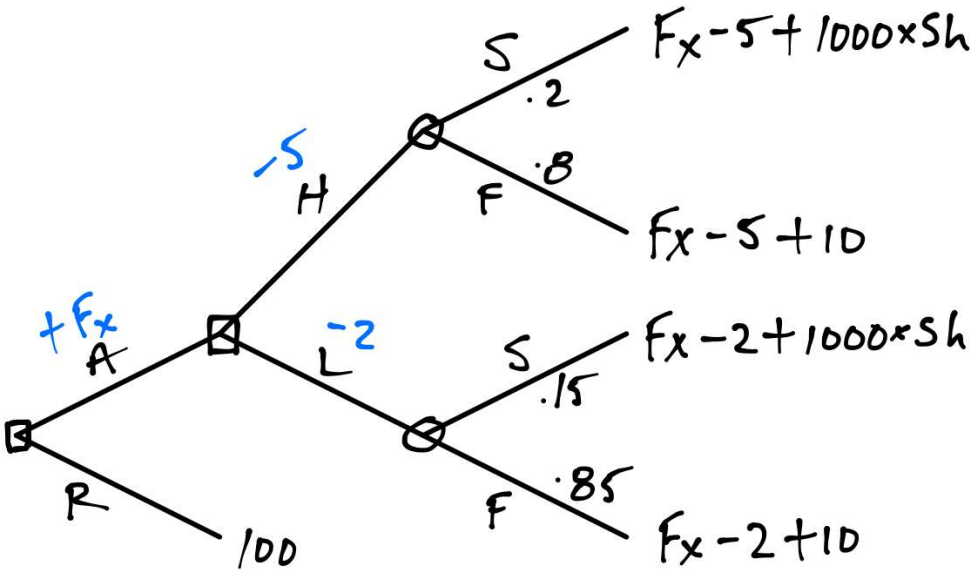
A, then L: $EV_L =$ 98
R: $EV_N =$ **100** Does not participate

Double fail:

1	F would pick L, not H	Not *incentive-compatible* (IC)
2	F wouldn't take contract	Fails *participation condition* (PC)

Case 2: Designing a contract that works

Founder's tree with **Fx** and **Sh** as variables:



F's payoffs from effort choice, in thousands:

$$EV_H = 0.2 * (Fx - 5 + 1000 * Sh) + 0.8 * (Fx - 5 + 10)$$

$$EV_H = Fx + 200 * Sh + 3$$

$$EV_L = 0.15 * (Fx - 2 + 1000 * Sh) + 0.85 * (Fx - 2 + 10)$$

$$EV_L = Fx + 150 * Sh + 6.5$$

$$EV_N = 100$$

IC: What's required for incentive compatibility?

- Want F to choose H: need $EV_H > EV_L$

$$Fx + 200 * Sh + 3 > Fx + 150 * Sh + 6.5$$

$$200 * Sh + 3 > 150 * Sh + 6.5$$

$$50 * Sh > 3.5$$

$$Sh > \frac{3.5}{50}$$

Conclusion: Sh > 7%

Need at least 7% ownership to have enough skin in the game

PC: What's required for participation?

- Want payoff from H to beat salary: need $EV_H > EV_N$

$$Fx + 200 * Sh + 3 > 100$$

$$Fx > 97 - 200 * Sh$$

One possible offer:

VC chooses: $Sh = 10\%$

$$Fx > 97 - 200 * 0.1$$

$$Fx > 77$$

Viable offer: $Fx = 80$

Does it work for the founder?

$$EV_H = 80 + 200 * (0.1) + 3 = 103$$

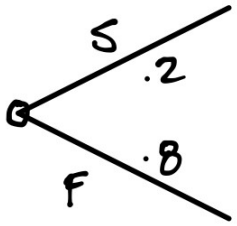
$$EV_L = 80 + 150 * (0.1) + 6.5 = 101.5$$

- Passes IC test: $EV_H > EV\_L$

$$EV_N = 100$$

- Passes PC test: $EV_H > EV_N$
- Net gain: 103 - 100 = 3

Is it OK for the VC?



$-Fx + (1 - Sh)*1000$

$-Fx$

General EV:

$$EV_V = 0.2 * (-Fx + (1 - Sh) * 1000) + 0.8 * (-Fx)$$

$$EV_V = -Fx + 200 * (1 - Sh)$$

This offer:

$$EV_V = -80 + 200 * 0.9$$

$$EV_V = -80 + 180 = 100$$

- Positive for the VC

Overall payoff:

Founder:	3k
VC:	100k
Total:	103k

What happened to the other 5k?