

E: Efficient incentive design, part 2

Case 2: Designing a contract that works

Two parameters:

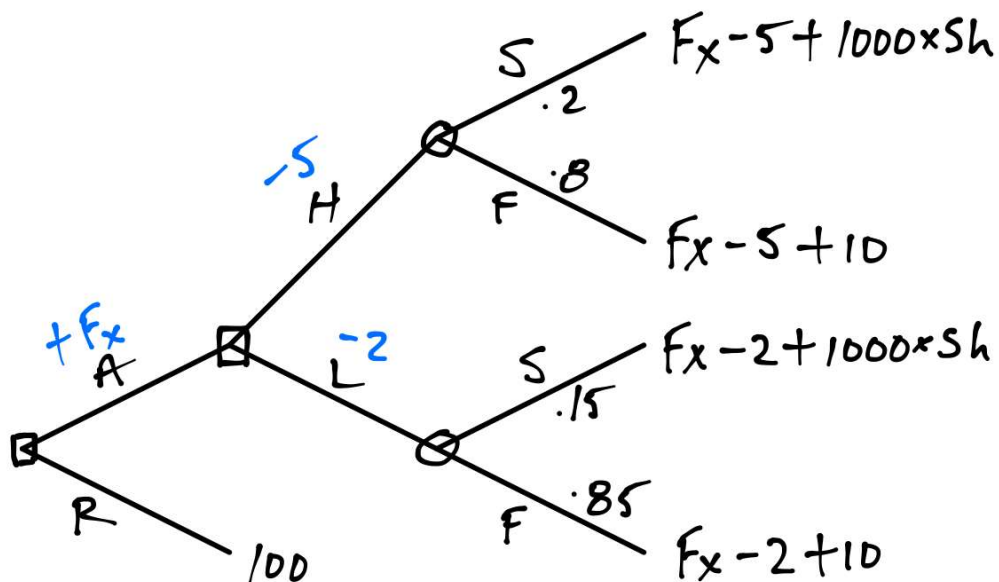
Fixed payment (Fx)

VC pays to F

Share of ownership (Sh)

Retained by F

Founder's tree with Fx and Sh variables:



F's payoffs from effort choice, in thousands:

$$EV_H = 0.2 * (Fx - 5 + 1000 * Sh) + 0.8 * (Fx - 5 + 10)$$

$$EV_H = Fx + 200 * Sh + 3$$

$$EV_L = 0.15 * (Fx - 2 + 1000 * Sh) + 0.85 * (Fx - 2 + 10)$$

$$EV_L = Fx + 150 * Sh + 6.5$$

$$EV_N = 100$$

IC: What's required for incentive compatibility?

- Want F to choose H: need $EV_H > EV_L$

$$Fx + 200 * Sh + 3 > Fx + 150 * Sh + 6.5$$

$$200 * Sh + 3 > 150 * Sh + 6.5$$

$$50 * Sh > 3.5$$

$$Sh > \frac{3.5}{50}$$

Conclusion: $Sh > 7\%$

Need at least 7% ownership to have enough skin in the game

PC: What's required for participation?

- Want payoff from H to beat salary: need $EV_H > EV_N$

$$Fx + 200 * Sh + 3 > 100$$

$$Fx > 97 - 200 * Sh$$

One possible offer:

VC chooses: $Sh = 10\%$

$$Fx > 97 - 200 * 0.1$$

$$Fx > 77$$

Viable offer: $Fx = 80$

Does it work for the founder?

$$EV_H = 80 + 200 * (0.1) + 3 = 103$$

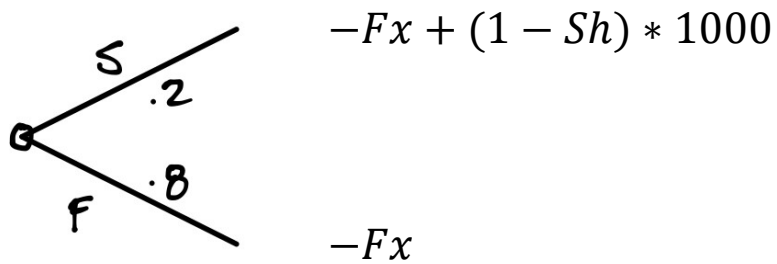
$$EV_L = 80 + 150 * (0.1) + 6.5 = 101.5$$

- Passes IC test: $EV_H > EV_L$

$$EV_N = 100$$

- Passes PC test: $EV_H > EV_N$
- Net gain: $103 - 100 = 3$

Is it OK for the VC?



General EV:

$$EV_V = 0.2 * (-Fx + (1 - Sh) * 1000) + 0.8 * (-Fx)$$

$$EV_V = -Fx + 200 * (1 - Sh)$$

This offer:

$$EV_V = -80 + 200 * 0.9$$

$$EV_V = -80 + 180 = 100$$

- Positive for the VC

Overall payoff:

Founder: 3k

VC: 100k

Total: 103k

What happened to the other 5k?

Generalizing to find range of viable contracts:

Founder:

$$\text{IC: } EV_H > EV_L \quad Sh > 7\%$$

$$\text{PC: } EV_H > EV_N \quad Fx > 97 - 200 * Sh$$

VC:

$$\text{VC: } EV_V > 0 \quad 200 * (1 - Sh) > Fx$$

Graphing:

Sh on Y, Fx on X

Intercepts:

$$\text{IC: } Sh > 7\%$$

$$Y : 7\%$$

$$X : na$$

Also, higher Sh is better

$$\text{PC: } Fx > 97 - 200 * Sh$$

$$X : Sh = 0, Fx = 97$$

$$Y : Fx = 0, Sh = 48.5\%$$

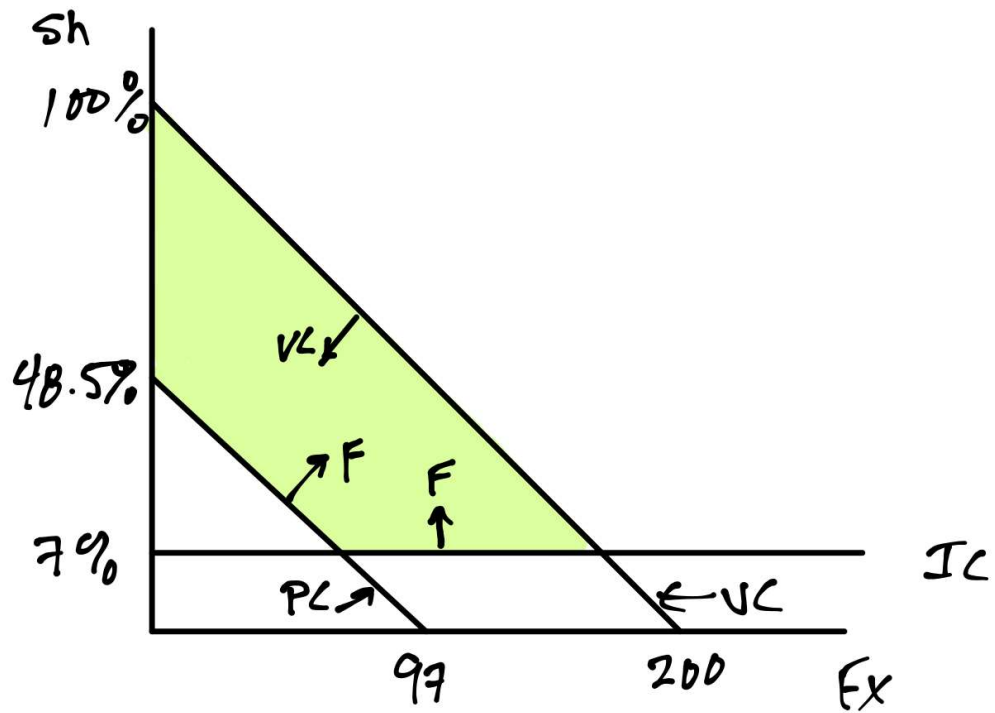
Also, higher Fx or Sh is better

$$\text{VC: } 200 * (1 - Sh) > Fx$$

$$X : Sh = 0, Fx = 200$$

$$Y : Fx = 0, Sh = 100\%$$

Also, lower Sh or Fx is better



For reference: intersection of IC, PC
 $Sh = 7\%, Fx = 83k$