E: Efficient incentive design, part 2

Summary of the principal-agent (PA) version of the biofuels startup

Participants:

Founder (F):	Has **idea** but no cash
Venture capitalist (VC):	Has **cash** but no idea

Payoffs:

Success (S):	$1M
Failure (F):	$10k
No startup (N):	$100k

Founder's effort (E) affects chance of success (S) but is costly to F:

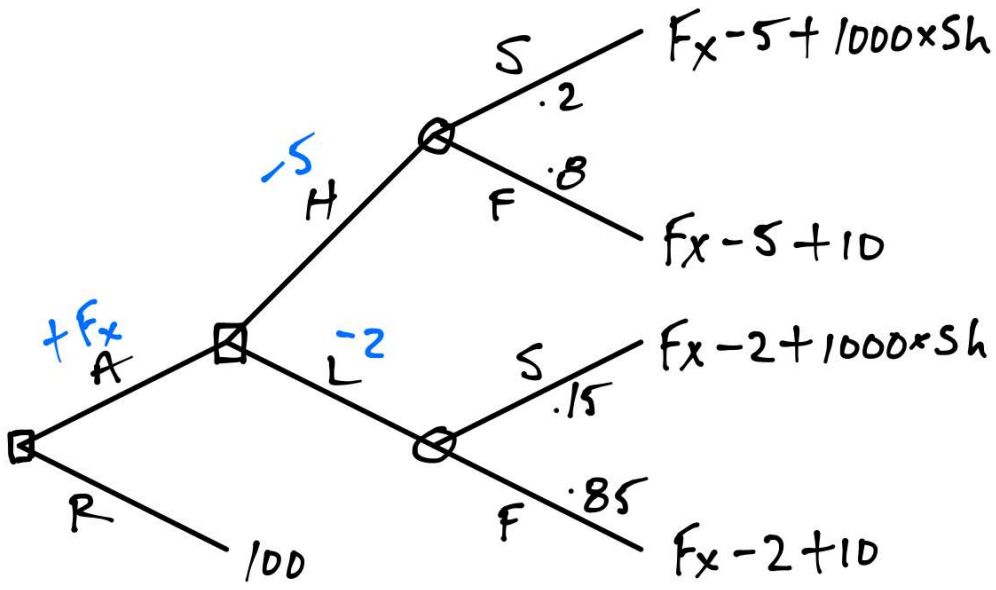
Level of effort	Cost to F	Prob of S
High (H):	$5k	20%
Low (L):	$2k	15%

Case 2: Designing a contract that works

Two parameters:

Fixed payment (**Fx**)	VC pays to F
Share of ownership (**Sh**)	Retained by F

Founder's tree with **Fx** and **Sh** variables:



F's payoffs from effort choice, in thousands:

$$EV_H = 0.2 * (Fx - 5 + 1000 * Sh) + 0.8 * (Fx - 5 + 10)$$

$$EV_H = Fx + 200 * Sh + 3$$

$$EV_L = 0.15 * (Fx - 2 + 1000 * Sh) + 0.85 * (Fx - 2 + 10)$$

$$EV_L = Fx + 150 * Sh + 6.5$$

$$EV_N = 100$$

IC: What's required for incentive compatibility?

- Want F to choose H: need $EV_H > EV_L$

$$Fx + 200 * Sh + 3 > Fx + 150 * Sh + 6.5$$

$$200 * Sh + 3 > 150 * Sh + 6.5$$

$$50 * Sh > 3.5$$

$$Sh > \frac{3.5}{50}$$

Conclusion: Sh > 7%

Need at least 7% ownership to have enough skin in the game

PC: What's required for participation?

- Want payoff from H to beat salary: need $EV_H > EV_N$

$$Fx + 200 * Sh + 3 > 100$$

$$Fx > 97 - 200 * Sh$$

One possible offer:

VC chooses: $Sh = 10\%$

$$Fx > 97 - 200 * 0.1$$

$Fx > 77$

Viable offer: $Fx = 80$

Does it work for the founder?

$EV_H = 80 + 200 * (0.1) + 3 = 103$

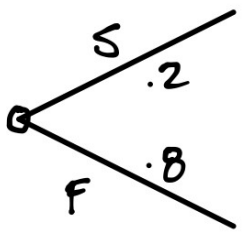
$EV_L = 80 + 150 * (0.1) + 6.5 = 101.5$

- Passes IC test: $EV_H > EV\_L$

$EV_N = 100$

- Passes PC test: $EV_H > EV_N$
- Net gain: 103 - 100 = 3

Is it OK for the VC?



$-Fx + (1 - Sh) * 1000$

$-Fx$

General EV:

$EV_V = 0.2 * (-Fx + (1 - Sh) * 1000) + 0.8 * (-Fx)$

$EV_V = -Fx + 200 * (1 - Sh)$

This offer:

$$EV_V = -80 + 200 * 0.9$$

$$EV_V = -80 + 180 = 100$$

- Positive for the VC

Overall payoff:

Founder:	3k
VC:	100k
Total:	103k

What happened to the other 5k?

Generalizing to find range of viable contracts:

Founder:

IC: $EV_H > EV_L$ $Sh > 7\%$

PC: $EV_H > EV_N$ $Fx > 97 - 200 * Sh$

VC:

VC: $EV_V > 0$ $200 * (1 - Sh) > Fx$

Graphing:

Sh on Y, Fx on X

Intercepts:

IC: $Sh > 7\%$

Y : 7%
X : na

Also, higher Sh is better

PC: $Fx > 97 - 200 * Sh$

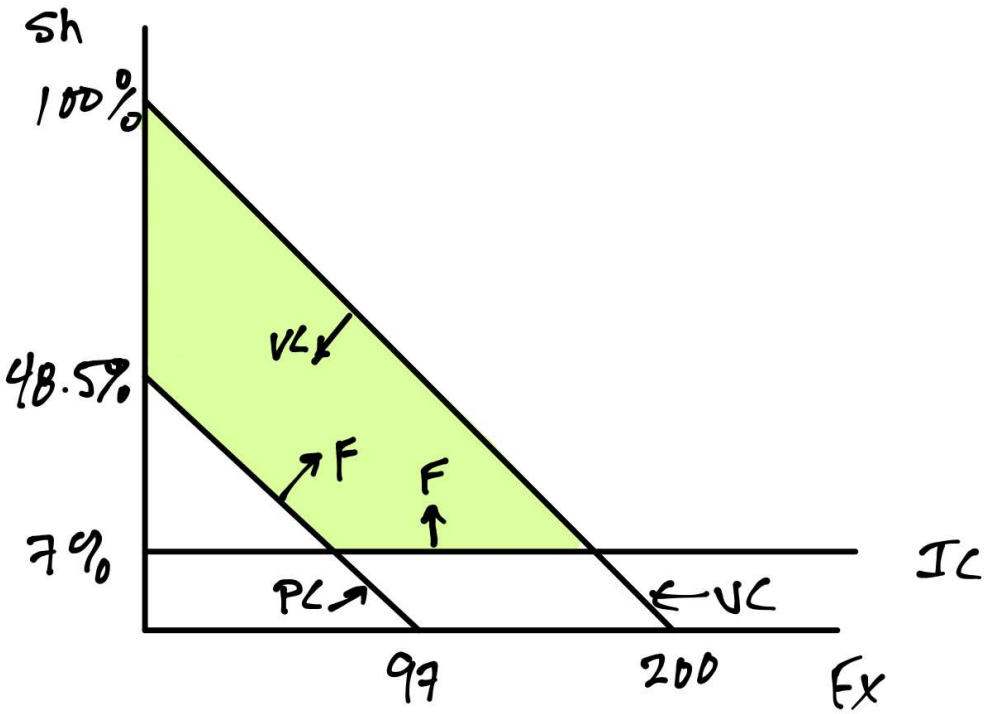
X : $Sh = 0, Fx = 97$
Y : $Fx = 0, Sh = 48.5\%$

Also, higher Fx or Sh is better

VC: $200 * (1 - Sh) > Fx$

X : $Sh = 0, Fx = 200$
Y : $Fx = 0, Sh = 100\%$

Also, lower Sh or Fx is better



For reference: intersection of IC, PC

$Sh = 7\%, Fx = 83k$