Cobb Douglas Preferences

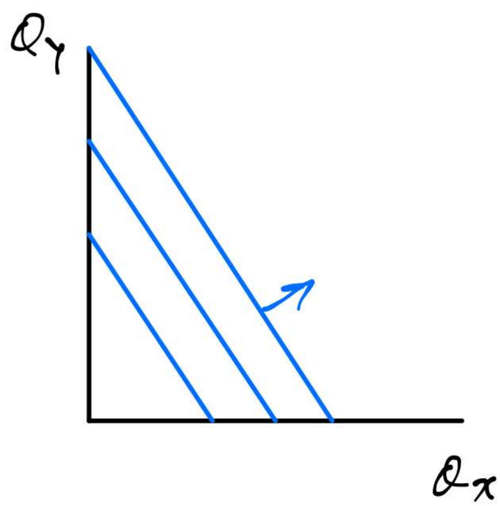
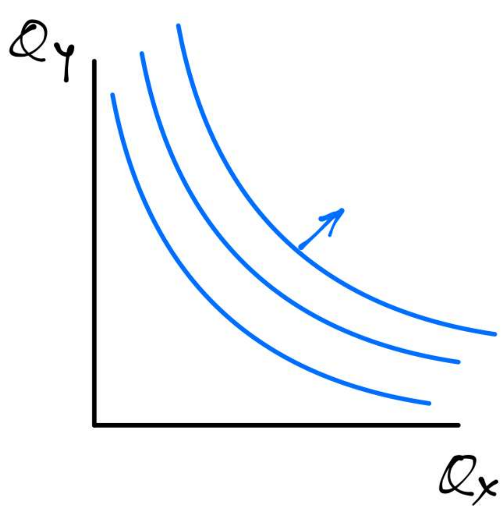
General Cobb-Douglas utility function:

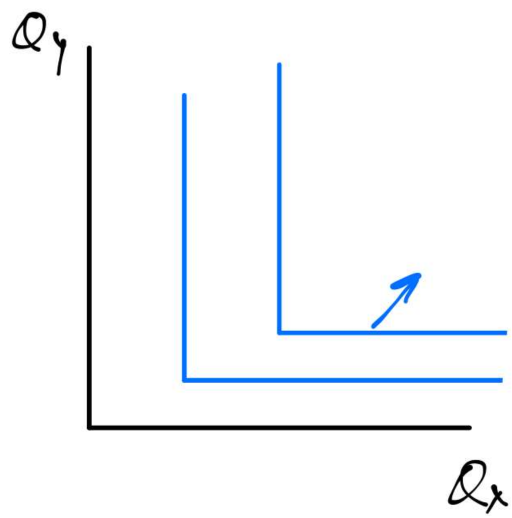
$$U = (Q_x)^a (Q_y)^{1-a}$$

Has downward-sloping curved ICs

a is a *parameter*: chosen to fit the specific case at hand

Conceptually, Cobb-Douglas is half way between previous extremes:

Type	Typical IC diagram	Characteristics
Perfect Substitutes (PS)		Perfectly willing to trade X for Y MRS is constant Extremes (only X or Y) are OK
Cobb-Douglas (CD)		Moderately willing to trade X for Y MRS varies Moderately prefers mixtures to extremes

Perfect Complements (PC)		Completely unwilling to trade X for Y No MRS Extremes are not OK: insists on a precise mixture of X and Y

Finding the CD demand equations

Using calculus and the CD utility function, can show slope of ICs is:

$$m = MRS = -\frac{aQ_y}{(1-a)Q_x}$$

Now have two equations to find the equilibrium:

Budget constraint:

$$P_xQ_x + P_yQ_y = M$$

Matching slopes of the IC and BC:

$$MRS = -\frac{P_x}{Py}$$

$$-\frac{aQ_y}{(1-a)Q_x} = -\frac{P_x}{P_y}$$

Solving two equations for Q_x and Q_y gives the CD demands:

$$Q_x = \frac{aM}{P_x} \qquad\qquad Q_y = \frac{(1-a)M}{P_y}$$

Interpretation of parameter a

Rearrange the demands:

$$\frac{P_x Q_x}{M} = a \qquad\qquad \frac{P_y Q_y}{M} = 1-a$$

a = share of M spent on X

$1-a$ = share of M spent on Y